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LRS Bianchi Type-I Cosmological Model with Cosmological constant and Quadratic EoS Parameter in $f(R,T)$ Gravity.

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Abstract:

In this paper ,we have studied LRS Bianchi type-I cosmological model employing the quadratic equation of state and considering the influence of cosmological constant to understand the nature of universe in $f(R,T)$ gravity theory. We studied a class of functional form, $f(R,T)=R+f(T)$ within the framework of $f(R,T)$ gravity theory and used a quadratic equation of state to study the models behaviour.

Key aspects: LRS Bianchi Type-I cosmological model, f (R, T) theory of gravity, Quadratic Equation of state, Cosmological Constant, Evolution of the universe.

1. Introduction

The study of cosmological models has been an active area of research in modern astrophysics and cosmology. One of the most popular and well-studied models is the Bianchi Type-I cosmological model, which is a spatially homogeneous and anisotropic model that describes the evolution of the universe. In recent years, there has been a growing interest in modifying the general theory of relativity to explain the observed phenomena of the universe, such as the accelerating expansion of the universe.

Observations from diverse sources, including Supernovae type-Ia experiments [1,2,3,4,5,6], cosmic microwave background fluctuations [7], baryon acoustic oscillations [8,9,10], and PLANK collaborations [11], consistently indicate that the universe is expanding at an accelerating rate. To explain this phenomenon, researchers have proposed two main approaches: introducing a dark energy component or modifying the general theory of relativity. The simplest modification involves adding a cosmological constant Λ to Einstein's field equations, but this approach is plagued by fine-tuning and cosmic coincidence problems. As a result, alternative modifications have been explored, with the f(R, T) modified gravity theory gaining significant attention.

The f (R, T) theory of gravity, proposed by Harko et al. [12], is one such modification of general relativity, where R is the Ricci scalar and T is the trace of the energy-momentum tensor. This theory has been widely used to study various cosmological models, including the Bianchi Type-I model. Many researchers have considered many problems of the cosmology in this theory. Adhav [13] has developed LRS Bianchi Type-I Space time filled with perfect fluid in f (R, T) gravity. The perfect fluid solutions of Bianchi type-I space time in scalar tensor theory have been explored by Kumar and Singh [14], In the framework of f(R; T) gravity, Sahoo, P.K. and Sivakumar, M.[15] explained LRS Bianchi type-I cosmological model in f(R, T) theory of gravity with Λ(T), P. Shukla et. al. [16] studied LRS Bianchi Type-I Cosmology with Gamma Law EoS in f(R, T) Gravity, Reddy, D. R. K., et. al. [17] have investigated Bianchi type-III Dark Energy Model in f(R, T) Gravity, Sahoo, P. et. al. [18] discussed the locally rotationally symmetric (LRS) Bianchi type-I cosmological model in f (R, T) gravity with bulk viscous fluid as matter content. Ladke, L. S., et. al.[19] studied Bianchi type-III cosmological model with $f(R,T)$ gravity in the presence of perfect fluid based on Lyra geometry. Bhardwaj et. al. [20] investigated the cosmological bouncing solution in LRS Bianchi-I space-time in framework of $f(R,T)$ gravity, Locally rotational symmetric Bianchi Type-II cosmological model is studied using the framework of f (R, T) gravity with a non-linear form of f (R, T) function analyzed by Nisha Godani [21]. R. K. Tiwari, et. al. [22] explored Bianchi Type III String Cosmological Model in $f(R,T)$ Modified Gravity Theory. D.T. Solanke et. al.[23] presented the interacting scalar and electromagnetic fields in Bianchi type I space–time in $f(R,T)$ theory of gravity. Mahanta et. al. [24] have explored Bianchi Type-V Universe with time varying cosmological constant and quadratic equation of state in $f(R,T)$ theory of gravity. S. Ayugun et. al. [25] have investigated Quadratic equation of state solutions with in $f(R,T)$ gravitation theory. The perfect fluid matter distribution using three Quadratic equation of states for plane symmetric space time in $f(R,T)$ gravity have been studied by Ladke, L. S., et. al. [26]

Recently, Rej, Pramit et. al. [27] studied "Traversable dark energy wormholes in a quadratic f $f(R,T)$ gravity, K.Pawar, N.T. Katre [28] observes Anisotropic Bianchi Type-V Perfect Fluid Cosmological Models in the frame work of $f(R,T)$ Gravity. Rishi Kumar Tiwari et. al. [29] examined the Time Varying Deceleration Parameter in f(R, T) Gravity.A spatially homogeneous and anisotropic locally-rotationally-symmetric Bianchi type-I spacetime model with strange quark matter (SQM) and a variable cosmological term Λ(t) and with constant expansion rate is studied in $f(R,T)$ theory by Singh, Vijay, and Aroonkumar Beesham [30,31], V. A. Thakare, R. V. Mapari, and S. S. Thakre.[32] obtained "Five-Dimensional Plane Symmetric Cosmological Model with Quadratic Equation of State in $f(R,T)$ Theory of Gravity. Ladke, L. S., et. al. [33] obtained Five-dimensional exact solutions of Bianchi type-I space-time in $f(R,T)$ theory of gravity.

In this context, we investigate the LRS Bianchi Type-I cosmological model in the presence of a quadratic equation of state with a cosmological parameter in the $f(R,T)$ theory of gravity. The quadratic equation of state is a more general form of the equation of state, which can describe a wide range of cosmic fluids, including dark energy and dark matter.

Our study aims to explore the cosmological implications of the LRS Bianchi Type-I model in the $f(R,T)$ theory of gravity, with a focus on the effects of the quadratic equation of state and the cosmological parameter on the evolution of the universe. This paper organized as follows: In section (2), we consider the Bianchi Type-I space time and developed a Einstein's field equations in f (R, T) theory of gravity.

Sections (3) and (4) is used to find solution of anisotropic Bianchi Type-I space time filled with perfect fluid along with some physical quantities. In last section we discuss the conclusion.

Gravitational Field Equations of $f(R,T)$ Gravity

The action of the modified $f(R,T)$ gravity can be given as

$$S=\int \frac{f(R,T)}{16\pi G}\sqrt{-g}\,d^4x+\int L_m\sqrt{-g}\,d^4x \qquad (1)$$

Where R is the Ricci scalar, T is the trace of energy momentum tensor and $f(R,T)$ is an arbitrary function of R and T. L_m is the matter Lagrangian density.

We are interesting to study the problem with the inclusion of the cosmological parameter Λ of its physical importance in the discussions of DE and DM.

Hence varying the action S with respect to the metric tensor $g_{\alpha\beta}$, the field equations of $f(R,T)$ gravity with cosmological parameter Λ are given by

$$f_R(R,T)R_{\alpha\beta}-\frac{1}{2}f(R,T)g_{\alpha\beta}-(\nabla_\alpha\nabla_\beta-g_{\alpha\beta}\Box)f_R(R,T)=8\pi T_{\alpha\beta}-f_T(R,T)T_{\alpha\beta}-f_T(R,T)\Theta_{\alpha\beta}+\Lambda g_{\alpha\beta} \qquad (2)$$

Where $\Box\equiv\nabla^i\nabla_i$ is the De Alembert's operator, f_R and f_T are the ordinary derivatives with respect to Ricci scalar R and trace of energy momentum tensor T respectively.

Contraction of equation (2) gives

$$f_R(R,T)R+3\Box f_R(R,T)-2f(R,T)=8\pi T-Tf_T(R,T)-f_T(R,T)\Theta+\Lambda \qquad (3)$$

Where $\Theta=g^{ij}\Theta_{ij}=\Theta_i^i$.

The stress-energy tensor of the matter Lagrangian is

$$T_{\alpha\beta}=(\rho+p)u_\alpha u_\beta-pg_{\alpha\beta} \qquad (4)$$

The perfect fluids described by an energy density ρ and pressure p. The four velocity $u^i=(1,0,0,0)$ in co-moving co-ordinates satisfying the condition $u_iu^i=1$ and $u_i\nabla_j u^i=0$. Thus we can assume $L_m=-p$, which yields

$$\Theta_{\alpha\beta}=-2T_{\alpha\beta}-p\,g_{\alpha\beta} \qquad (5)$$

Singh and Bishi [34] have discussed Bianchi I transit space–time for $f(R,T)$ theory with cosmological constant Λ and Reddy et al. [35] have obtained quadratic EoS solutions in GR for Bianchi-I universe in the following form of quadratic EoS

$$p = \varepsilon \rho^2 - \rho \qquad (6)$$

Here $\varepsilon \neq 0$ is constant

Using equations (5), the field equations (2) can be written as

$$f_R(R,T)R_{\alpha\beta} - \frac{1}{2} f(R,T) g_{\alpha\beta} - (\nabla_\alpha \nabla_\beta - g_{\alpha\beta}\Box) f_R(R,T) = 8\pi T_{\alpha\beta} + f_T(R,T) T_{\alpha\beta} + p f_T(R,T) g_{\alpha\beta} + \Lambda g_{\alpha\beta} \qquad (7)$$

Using the functional form of field equations

$$f(R,T) = R + 2f(T). \qquad (8)$$

Where $f(T)$ is an arbitrary function of stress energy tensor of matter.

Here we consider $f(T) = \mu T$ *and* $f'(T) = {df}/{dT}$

Where μ is a constant.

With help of Eqns (8), the gravitational field equations of $f(R,T)$ gravity with cosmological parameter Λ can be expressed as

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = 8\pi T_{\alpha\beta} + 2f'(T) T_{\alpha\beta} + [\Lambda + f(T) + 2p\, f'(T)] g_{\alpha\beta} \qquad (9)$$

Metric and Field Equations for $f(R,T) = R + 2f(T)$ with cosmological Parameter Λ

Considering the line element of LRS Bianchi type-I space-time

$$ds^2 = dt^2 - A^2 dx^2 - B^2(dy^2 + dz^2) \qquad (10)$$

Where A *and* B cosmic scale factors are function of t only.

The corresponding Ricci scalar is obtained as

$$R = 4\left[\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB}\right] + 2\left[\frac{\dot{B}^2}{B^2} + \frac{\ddot{A}}{A}\right] \qquad (11)$$

Where overhead (.) denotes derivative with respect to time t.

Now we define spatial volume

$$V = AB^2 \qquad (12)$$

The directional Hubble's parameters

$H_1 = \frac{\dot{A}}{A}$ *and* $H_3 = H_2 = \frac{\dot{B}}{B}$ in the x. y and z axes direction respectively.

The generalized mean Hubble's parameter is given as

$$H = \frac{1}{3}\left[\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right] \tag{13}$$

The expansion scalar is given by

$$\theta = \left[\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right] \tag{14}$$

The shear scalar and the mean anisotropic parameter is define as

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \tag{15}$$

and

$$A_m = \frac{1}{3H^2}\left[\sum_{i=1}^{3} H_i^2 - 3H^2\right] \tag{16}$$

Deceleration parameter is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{17}$$

Now using field equations (9) with the help of equation (4) for the metric (10), it gives

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = -8\pi\rho + p\mu - 3\mu\rho - \Lambda \tag{18}$$

$$-\frac{2\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} = -8\pi p - 3p\mu + \mu\rho + \Lambda \tag{19}$$

$$-\frac{\ddot{A}}{A} - \frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{B}}{B} = -8\pi p - 3p\mu + \mu\rho + \Lambda \tag{20}$$

These are the three non-linear independent equations with five unknowns namely A, B, p, ρ *and* Λ.

Solutions of the Field Equations

Equations (18)-(20), reduce to two independent field equations

$$-\frac{2\ddot{B}}{B} - \frac{2\dot{A}\dot{B}}{AB} - \frac{2\dot{B}^2}{B^2} = 8\pi(\rho - p) + 4\mu(\rho - p) + 2\Lambda \tag{21}$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{3\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} = 8\pi(\rho - p) + 4\mu(\rho - p) + 2\Lambda \tag{22}$$

Subtracting equation (22) from equation (21)

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}^2}{B^2}=0 \qquad (23)$$

Solving equation (23), we get

$$\frac{A}{B}=d_2 e^{\int \frac{d_1}{a^3}dt} \qquad (24)$$

Where d_1 and d_2 are constants of integration.

Subtract equation (21) from equations (22)

$$\frac{\ddot{B}}{B}-\frac{\ddot{A}}{A}-\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}=0 \qquad (25)$$

Solving this equation

$$\frac{B}{A}=d_4 e^{\int \frac{d_3}{a^3}dt} \qquad (26)$$

Where d_3 and d_4 are constants of integration.

Solving equations (24) & (26), we can write the metric functions explicitly as

$$A=a\, p_1\, e^{\int \frac{q_1}{a^3}dt} \qquad (27)$$

$$B=a\, p_2\, e^{\int \frac{q_2}{a^3}dt} \qquad (28)$$

Where $p_1=(d_2^2 d_4^4)^{-1/3}$, $p_2=(d_2 d_4^2)^{1/3}$ and $q_1=\frac{-(2d_1+4d_3)}{3}$, $q_2=\frac{d_1+2d_3}{3}$

The deceleration parameter q in cosmology is the measure of the cosmic acceleration of the universe expansion & is defined as

$$\dot{a}=la^{-m+1} \qquad \ddot{a}=-l^2(m-1)a^{-2m+1}$$

Substituting the values of $\dot{a}$ & $\ddot{a}$ in equations (17), we get

$$q=m-1 \quad , \quad m=q+1 \qquad (29)$$

We also use well known relation between the average Hubble parameter H and average scale factor a given as

$$H = la^{-m} \tag{30}$$

Where $l > 0$ & $m \neq 0$ are constant.

Solving equations (30) we get values of average scale factor

$$a = (mlt + c)^{\frac{1}{m}} \tag{31}$$

Where c are integration constant.

Using equations (31) in equations (27) and (28), we obtained

The relation between cosmic scale factor and average scale factor

$$AB^2 = a^3 = (mlt + c)^{\frac{3}{m}} \tag{32}$$

This satisfy the relations

$$p_1 p_2^2 = 1 \qquad q_1 + 2q_2 = 0$$

Using equations (31), we get values of A and B

$$A = (mlt + c)^{\frac{1}{m}} p_1 e^{\int \frac{q_1}{(mlt+c)^{\frac{3}{m}}} dt} \tag{33}$$

$$B = (mlt + c)^{\frac{1}{m}} p_2 e^{\int \frac{q_2}{(mlt+c)^{\frac{3}{m}}} dt} \tag{34}$$

After simplifying, equations (33) and (34), turn out as

$$A = (mlt + c)^{\frac{1}{m}} p_1 e^{\frac{q_1 (mlt+c)^{m-3/m}}{l(m-3)}} \tag{35}$$

$$B = (mlt + c)^{\frac{1}{m}} p_2 e^{\frac{q_2 (mlt+c)^{m-3/m}}{l(m-3)}} \tag{36}$$

Physical Characteristics of the universe

Using equations (18) and (19) with the help of Eos parameters given in equations (6), we get energy density of the fluid as

$$\rho^2 = \frac{1}{(4\pi + 2\mu)\varepsilon}\left[\frac{q_2^2}{(mlt+c)^{6/m}} - \frac{(q_1+q_2)}{(mlt+c)^{3/m}} - \frac{q_1 l}{(mlt+c)^{3+m/m}} - \frac{l^2}{(mlt+c)^2} - \frac{l^2(1-m)}{(mlt+c)^{2+2m/m}}\right] \quad (37)$$

From equations (21) and equations (37), we get cosmological constant Λ is

$$\Lambda = \left[\frac{3q_2 l}{(mlt+c)^{3+m/m}} - \frac{l^2}{(mlt+c)^2} - \frac{l^2(1-m)}{(mlt+c)^{2+2m/m}}\right] \quad (38)$$

Using equations (37) and (38) in equations (21), we get pressure of the fluid

$$p = \frac{1}{(4\pi + 2\mu)}\left[\frac{3q_2 l}{(mlt+c)^{3+m/m}} - \frac{l^2}{(mlt+c)^2} - \frac{l^2(1-m)}{(mlt+c)^{2+2m/m}}\right] \quad (39)$$

The spatial volume of the universe is

$$V = (mlt+c)^{\frac{3}{m}} \quad (40)$$

The generalising mean Hubble parameter H is given as

$$H = \frac{l}{(mlt+c)} \quad (41)$$

The expansion scalar is found to be

$$\theta = \frac{3l}{(mlt+c)} \quad (42)$$

The shear scalar and the mean anisotropic parameter is given as

$$\sigma^2 = \frac{(q_1 - q_2)^2}{3(mlt+c)^{6/m-2}} \quad (43)$$

and

$$A_m = \frac{2(q_1 - q_2)^2}{9l^2(mlt+c)^{6/m-2}} \quad (44)$$

Conclusion:

Here we have studied LRS Bianchi type-I cosmological model employing the quadratic EOS and considering the influence of cosmological constant to understand the nature of universe in $f(R,T)$ gravity theory. We studied a class of functional form $f(R,T) = R + f(T)$ within the $f(R,T)$ gravity framework and used a quadratic EOS to study the models behaviour, we have following result.

Expansion scalar θ which measure the rate of expansion is finite at $t = 0$ and decreasing when t increases. It indicates that expansion diminishes as time passes. From the equation of spatial volume it is observed that universe expanding. Generalizing mean Hubble parameter has finite positive value, this shows an expanding universe and as time tends to infinity, it slows and tends to zero. This shows that expansion of the universe is fast during early period and slowing as time increases. From the equation of shear scalar, it is observed that, shear scalar is finite at initial stage, while shear scalar tending to zero as time approaches to infinity i.e. $\sigma^2 \to 0$ as $t = \infty$.This indicates that anisotropy of the universe tending towards isotropy over time. Anisotropy parameter shows that as time passes it diminishes. This shows that the universe was in anisotropic phase and moves towards isotropy when time increases our results are same as that of current observational data about the universe behaviour.

References

1. Riess, A.G., Filippenko, A.V., Challis, P., Clocchiatti, A., Diercks, A., Garnavich, P.M., et al. (1998) Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. The Astronomical Journal , 116, 1009-1038.
2. Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R.A., Nugent, P., Castro, P.G., et al. (1999) Measurements of Ω and Λ from 42 High-Redshift Supernovae. The Astronomical Journal, 517, 565-586.
3. Clocchiatti, A., Schmidt, B.P., Filippenko, A.V., Challis, P., Coi, A.L., Covarrubias, R., et al. (1999) Hubble Space Telescope and Ground-Based Observations of Type Ia Supernovae at Redshift 0.5: Cosmological Implications. The Astronomical Journal , 642, 1-21.
4. Spergel, D.N., Bean, R., Dore, O., Nolta, M.R., Bennett, C.L., Dunkley, J., et al . (2007) Three-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Implications for Cosmology. The Astrophysical Journal Supplement Series , 170, 377-408.
5. Riess, A.G., Kirshner, R.P., Schmidt, B.P., Jha, S., Challis, P., Garnavich, P.M., et al . (1999) BVRI Light Curves for 22 Type Ia Supernovae. The Astronomical Journal , 117, 707-724.
6. Knop, R.A., Aldering, G., Amanullah, R., Astier, P., Blanc, G., Burns, M.S., et al. (2003) New Constraints on ΩM, ΩΛ, and w from an Independent Set of 11 High-Redshift Supernovae Observed with the Hubble Space Telescope . The Astrophysical Journal , 598, 102-137.
7. Spergel, D.N., Verde, L., Peiris, H.V., Komatsu, E., Nolta, M.R., Bennett, C.L., et al . (2003) First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters. The Astrophysical Journal Supplement Series , 148, 175-194.
8. Nojiri, S., Odintsov, S.D. and Sami, M. (2006) Dark Energy Cosmology from Higher- Order, String-Inspired Gravity, and Its Reconstruction. Physical Review D, 74, Article ID: 046004.

9. Kamenshchik, A., Moschella, U. and Pasquier, V. (2001) An Alternative to Quintessence. Physics Letters B , 511, 265-268.
10. Martin, J. (2008) Quintessence: A Mini-Review. Modern Physics Letters A, 23, 1252-1265.
11. Ade, P.A.R., Aghanim, N., Arnaud, M., Ashdown, M., Aumont, J., Baccigalupi, C., et al . (2015) Planck 2015 Results. XIII. Cosmological Parameters. arXIV: 1502.01589.
12. Harko, T., Lobo, F.S.N., Nojiri, S. and Odintsov, S.D. (2011) f (R,T) Gravity. Physical Review D, 84, Article ID: 024020.
13. Adhav, K.S. (2012) LRS Bianchi Type-I Cosmological Model in f (R,T) Theory of Gravity. Astrophysics and Space Science , 339, 365-369.
14. Kumar, Suresh, and C. P. Singh. "Exact bianchi type-I cosmological models in a scalar-tensor theory." International Journal of Theoretical Physics 47 (2008): 1722-1730.
15. Sahoo, P. K., and M. Sivakumar. "LRS Bianchi type-I cosmological model in f (R, T) theory of gravity with Λ (T)." Astrophysics and Space Science 357 (2015): 1-12.
16. Shukla, Pratishtha, and Amritha Jayadev. "LRS Bianchi Type-I Cosmology with Gamma Law EoS in f (R; T) Gravity." Applications and Applied Mathematics: An International Journal (AAM) 11.1 (2016): 14.
17. Reddy, D. R. K., R. Santhi Kumar, and T. V. Pradeep Kumar. "Bianchi type-III dark energy model in f (R, T) gravity." International Journal of Theoretical Physics 52 (2013): 239-245.
18. Sahoo, Parbati, and Raghavender Reddy. "LRS Bianchi type-I bulk viscous cosmological models in f (R, T) gravity." Astrophysics 61.1 (2018): 134-143.
19. Ladke, L. S., B.V. Bansole, and V.P.Tripade. "Bianchi type-III cosmological model with $f(R,T)$ gravity based on Lyra geometry."Int. J. of Scientific Res. and Eng. Trends 11.2(2025):1987-1990.
20. Bhardwaj, Vinod Kumar, and Archana Dixit. "LRS Bianchi type-I bouncing cosmological models in f (R, T) gravity." International Journal of Geometric Methods in Modern Physics 17.13 (2020): 2050203.
21. Godani, Nisha. "LRS Bianchi Type II in f (R, T) gravity." AIP Conference Proceedings. Vol. 2253. No. 1. AIP Publishing, 2020.
22. Tiwari, R. K., Bhupendra K. Shukla, and Soma Mishra. "Bianchi Type III String Cosmological Model in f (R, T) Modified Gravity Theory." Prespacetime Journal 10.3 (2019).
23. Solanke, Dattarao T., and T. M. Karade. "Bianchi type I universe filled with scalar field coupled with electromagnetic fields in f (R, T) theory of gravity." Indian Journal of Physics 91 (2017): 1457-1466.
24. Mahanta, Chandra Rekha, Shayanika Deka, and Manash Pratim Das. "Bianchi type V universe with time varying cosmological constant and quadratic equation of state in f (R, T) theory of gravity." East European Journal of Physics 1 (2023): 44-52.
25. Aygün, S., C. Aktaş, and B. Mishra. "Quadratic equation of state solutions with\varLambda Λ in f (R, T) gravitation theory." Indian Journal of Physics 93 (2019): 407-414.
26. Ladke, L. S., S.D.Mhashakhetri, and V.P.Tripade. " Plane Symmetric Cosmological Model and Quadratic Equations of State Solutions in $f(R,T)$ Gravity . "Int. J. of Sci. Eng. and Techn. 13:2 (2025).
27. Rej, Pramit, Mustafa Salti, and Oktay Aydogdu. "Traversable dark energy wormholes in a quadratic f (R, T) gravity." Modern Physics Letters A (2025): 2550024.
28. Pawar, Kalpana, and N. T. Katre. "Anisotropic Bianchi Type-V Perfect Fluid Cosmological Models in f (R, T) Gravity." Int. J. Sci. Res. in Physics and Applied Sciences 11.4 (2023).

29. Tiwari, Rishi Kumar, Sushil Kumar Mishra, and Sateesh Kumar Mishra. "Time Varying Deceleration Parameter in f (R, T) Gravity." Journal of Applied Mathematics and Physics 9.5 (2021): 847-855.
30. Singh, Vijay, and Aroonkumar Beesham. "LRS Bianchi I model with strange quark matter and Λ (t) in f (R, T) gravity." New Astronomy 89 (2021): 101634.
31. Singh, Vijay, and Aroonkumar Beesham. "LRS Bianchi I model with constant expansion rate in f (R, T) gravity." Astrophysics and Space Science 365.7 (2020): 125.
32. Thakare, V. A., R. V. Mapari, and S. S. Thakre. "Five-Dimensional Plane Symmetric Cosmological Model with Quadratic Equation of State in f (R, T) Theory of Gravity." East European Journal of Physics 3 (2023): 108-121.

33. Ladke, L. S., V. K. Jaiswal, and R. A. Hiwarkar. "Five-dimensional exact solutions of Bianchi type-I space-time in f (R, T) theory of gravity." Int. J. Innov. Res. Sci. Eng. Techn.2014.3 8 (2014): 15332-15342.
34. Singh, G. P., and B. K. Bishi , Astrophys. Space Sci. 360(1) 34 (2015
35. Reddy, D. R. K., K. S. Adhav and M. A. Purandare ,Astrophys. Space Sci. 357(1) 20 (2015)