Advanced International Journal for Research (AIJFR)
E-ISSN: 3048-7641 ● Website: www.aijfr.com ● Email: editor@aijfr.com

Practical Formula for Finding the Square of Two-Digit Numbers Mentally

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Abstract

Our primary goal in this project was to instill a love of mathematics in middle school students and unlock its secrets. We reasoned that middle school students spend a long time calculating the squares of two-digit numbers and sometimes make simple mistakes. Students often make simple mistakes even when solving these problems with pencil and paper. In this process, we wanted to calculate the square of a two-digit number mentally, using a practical, fast, and accurate formula. We found examples and calculated the squares of two-digit numbers. We tried to find the simplest formula. While working on the examples, we realized there was an algorithm for finding the solution and formalized it. We discovered that formulas had already been developed. However, the new formula we developed had to be much simpler and more memorable than the old ones, avoiding complex calculations. Therefore, we used the nearest tenth to the number we were squaring, discovered a relatively simple algorithm, and developed a new formula. Our experiments revealed that previously used formulas were more complex, but ours was more practical, easy to calculate, avoided complex operations, and was more memorable. With our formula, we used simple operations to calculate the square of a two-digit number by rounding to the nearest ten. This made it possible to perform the square calculations mentally. This formula, which allows us to calculate the square of two-digit numbers without using paper and pencil, allowed us to perform these complex calculations using simple multiplication, addition, or subtraction of simple numbers.

Aim

Our primary goal in this project is to contribute to the science of mathematics and to demonstrate that some seemingly difficult calculations can actually be easily solved within mathematics, and that there may be many secrets within mathematics that have yet to be discovered or recognized. While calculating the square of two digits may seem difficult, we believe that this new formula we've developed will actually make it much easier than calculating it in our minds. We believe it could be a pioneering formula that can also be used to calculate the squares of numbers with three or more digits.

1. Introduction

Finding the square of two-digit numbers can be done with a simple multiplication. However, because the process involves multiplying two two-digit numbers, it can sometimes get complicated, and this requires a pencil and paper. When we began our research, we began by considering more practical ways to do this. Through our research, we discovered some formulas used on this subject. However, while these formulas may seem simple, we realized they weren't practical for mental calculations. We set out with the confidence that a more practical and memorable formula could be created.

Method

Our research goal was to find the square of two-digit numbers both practically and mentally. First, we examined existing methods. We realized that these formulas actually required impractical mental calculations. We began working on a new formula. The commonly used methods were as follows:

1- The first method is one of the most frequently used. According to this formula, if AB is a two-digit number, the square of AB can be calculated as AB^2; (where A = place value and a = number value).

$AB^2 = A^2 + 2AB + B^2$. For example, let's calculate the square of 43 together.

$39^2 = 30^2 + (2x30x9) + 9^2 = 900 + 540 + 81 = 1521$.

While the formula may seem simple, attempting to calculate it mentally can sometimes lead to serious computational difficulties and errors.

2- The second formula, while seemingly practical, is actually more complex. Accordingly, if AB is a two-digit number, the square of AB is calculated as AB^2;

AB + B = . Let's call this new digit AC. AC x a = . This is a three-digit number. Let's call this digit XYZ. Next, the square of the units digit of the number we initially planned to square is calculated as B^2, and let's call this digit Q. The result is $AB^2 = XYZ0 + Q$.

The logic behind this calculation is as follows:

Let our number be AB; (can be written as N = 10a + B)

Our number is AB = 10a + b. (Let a = place value, and a = number value)

$(10A+B)^2 = 100a^2 + 20ab + b^2 = 10a(10a + 2b) + b^2 = 10a(10a + b + b) + b^2 = 10a(N + b) + b^2$

As you can see, this formula is much more complex than the first one. Let's use it to calculate the square of the same number, as in the first example.

When we want to find 39^2:

Step 1: 39+9=48.

Step 2: 48x3=144

Step 3: 9x9=81

Step 4: 1440+81=1521.

As can be seen, this requires much more complex calculations than the first one. Mental calculations are often impractical if the numbers are not very convenient. For example, calculating 48x3 mentally won't be very easy. Similarly, if you keep this number in mind and then perform other calculations, the calculation becomes even more difficult.

3- Due to the impracticality of these measurements, we tried to find a new formula:

Our logic was based on finding the squares of numbers like 10, 20, 30, 80, and 90 easily.

To illustrate with an example:

20x20=400 (This also means the sum of 20 20's numbers)

20x19= (19 20 numbers, so subtracting 20 from 20x20 is enough to find this number) = 20x20 - 20= 400-20=380

19x19= (19 19 numbers) (Since we calculated 20 19 numbers above, subtracting one 19 from 20x19 gives us 19x19) 400-20-19=361

19x18= (18 19 numbers) (Then, if we subtract another 19 from the number above, we can easily find this number) 400-20-19-19=342

Thus, we realized that there is an algorithm in these calculations.

20x20=400

20x19=400-20

19x19=400-20-19=400-20-20+1

19x18=400-20-19-19

18x18=400-20-19-19-18=400-20-20-20-20+4

18x17=400-20-19-19-18-18

17x17=400-20-19-19-18-18-17=400-20-20-20-20-20-20+9

17x16=400-20-19 -19-18-18-17-17

16x16=400-20-19-19-18-18-17-17-16=400-20-20-20-20-20-20-20+16

16x15=400-20-19-19-18-18-17-17-16-16

15x15=400-20-19-19-18-18-17-17-16-16-15=400-20-20-20-20-20-20-20-20-20+25

If the nearest ten to the number we are trying to square is less than the number we are trying to square, perform the calculation in reverse. We translated:

10x10=100

11x10=100+10

11x11=100+10+11=100+10+10+1

12x11=100+10+11+11

12x12=100+10+11+11+12=100+10+10+10+10+4

13x12=100+10+11+11+12+12

13x13=100+10+11+11+12+12+13=100+10+10+10+10+10+10+9

14x13=10 0+10+11+11+12+12+13+13

14x14=100+10+11+11+12+12+13+13+14=100+10+10+10+10+10+10+10+10+16

15x14=100+10+11+11+12+12+13+13+14+14

15x15=100+10+11+11+12+12+13+13+14+14+15=100+10+10+10+10+10+10+10+10+10+10+10+25

3a) If the number whose square we are trying to calculate is greater than the number whose square we are trying to calculate to the nearest ten; (Squares of 19, 18, 17, 16, or 15)

We noticed that a unique algorithm has been created:

Let X be the nearest ten to the number. The number we are trying to square is Y.

Let X-Y=n.

20x20=400

19x19=400-20-19=400-20-20+1

18x18=400-20-19-19-18=400-20-20-20-20+4

17x17=400-20-19-19-18-18-17=400-20-20-20-20-20-20+9

16x16=400 -20-19-19-18-18-17-17-16=400-20-20-20-20-20-20-20-20+16

15x15=400-20-19-19-18-18-17-17-16-16-15=400-20-20-20-20-20-20-20-20-20-20+25

Example: Y=19 X=20 X-Y=n 20-19=1. n=1

19^2= 19x19=20^2 - 20 -20 +1= 20^2 - 2x20 + 1= X^2 - 2xX +n^2

18^2=18x18=20^2 - 20-20-20-20 +4 = 20^2 - 4xX + 4 = X^2 - 2nxX + n^2

Thus, if the nearest ten to the number whose square we are trying to find is greater than this number;

Y is the two-digit number whose square we are trying to find,

X is the nearest ten to the number whose square we are trying to find,

n=X-Y

Formula: Y^2=X^2 - 2nxX + n^2

3b) Now, if the nearest ten to the number whose square we are trying to find is less than the number whose square we are trying to find; (Squares of 11, 12, 13, 14, or 15)

We also noticed that a unique algorithm has been created;

Similar to the previous step, let X be the number whose square we are trying to calculate to the nearest ten. The number whose square we are trying to find is Y.

Let Y-X=n (This time, since the number we are trying to square is greater than the nearest ten, Y-X=n)

10x10=100

11x11=100+10+11=100+10+10+10+1

12x12=100+10+11+11+12=100+10+10+10+10+10+4

13x13=100+10+11+11+12+12+13=100+10+10+10+10+10+10+10+9

14x14 =100+10+11+11+12+12+13+13+14=100+10+10+10+10+10+10+10+10+16

15x15=100+10+11+11+12+12+13+13+14+14+15=100+10+10+10+10+10+10+10+10+10+10+25

Example: Y=11 X=10 Y-X=n 11-10=1. n=1

11^2= 11x11=10^2 + 10 +10 +1= 10^2 + 2x10 + 1= X^2 + 2xX +n^2

12^2=12x12=10^2 + 10+10+10+20 +4 = 10^2 - 4xX + 4 = X^2 + 2nxX + n^2

Thus, if the nearest ten to the number whose square we are trying to find is less than this number;

Y is the two-digit number whose square we are trying to find,

X is the nearest ten to the number whose square we are trying to find, If n=Y-X

Formula: $Y^2=X^2 + 2nxX + n^2$

Our formula in summary:

Y= The number we are trying to square.

X=The nearest ten to Y.

N is the difference between X and Y.

If the number we are trying to square is greater than the nearest ten;

$Y^2=X^2 - 2nxX + n^2$

If the number we are trying to square is less than the nearest ten;

$Y^2=X^2 + 2nxX + n^2$

The only difference between the two formulas is that if 2nX is greater than the number whose square we're trying to find to the nearest ten, it's subtracted from the square of X. If it's less than the number whose square we're trying to find to the nearest ten, it's added to the square of X.

As you can see, a very catchy formula emerges. Let's try finding the square of the number 39, which we've tried in other formulas before.

$39^2 = ?$

Y = 39. Therefore, since X = 40 to the nearest ten and 40 - 39 = 1, n = 1.

Then $39^2 = 40^2 - 2.1.40 + 1^2 = 1600 - 80 + 1 = 1521$.

While the first calculation example ($39^2 = 30^2 + (2x30x9) + 9^2 = 900 + 540 + 81 = 1521$) adds more complex numbers, our formula adds simple numbers like 1600, 80, and 1. Furthermore, the first formula must calculate 39 x 39, but our formula will not include such complex multiplications.

The second calculation example involved similar complex calculations.

(Step 1: 39+9=48.

Step 2: 48x3=144.

Step 3: 9x9=81.

Step 4: 1440+81=1521.)

Here, we also have some slightly complex calculations like 48x3.

However, our formula includes simple operations like 40x40, and operations like 40x2, which are also very simple to calculate mentally.

Now, let's do an example like 42, which is smaller than the number we're trying to square to the nearest ten:

42^2=?

Y=42. Therefore, since X = 40 and 42 - 40 = 2, n = 2.

Then $42^2 = 40^2 + 2.2.40 + 2^2 = 1600 - 160 + 4 = 1764$.

As you can see, since the calculations are always rounded to the nearest ten, they are simple enough to be done mentally. It's very easy to calculate the square of 40, 40 x 4, or the square of 2 mentally.

Let's multiply the examples and see if the calculation is practical:

For example, 85^2 = ? This calculation seems difficult at first glance. A particularly nice and practical aspect of the formula is that both formulas can be used when calculating the square of numbers whose ones digit is 5.

Solution 1: Y = 85. Then, since X=90 and 90-85=5, n=5

Then $85^2 = 90^2 - 2.5.90 + 5^2 = 8100 - 900 +25= 7225$

Solution 2: Y=85. Then, since X=80 and 85-80=5, n=5

Then $85^2 = 80^2 + 2,5,80 + 5^2 = 6400 + 800 +25= 7225$

Results;

Based on this data, the formula we developed is much more practical than the old formulas, allowing us to work with numbers that can be easily calculated mentally. When we gave students these three formulas, including our own, and asked them to calculate the squares of some two-digit numbers mentally, we found that those who calculated with our formula consistently found earlier and more accurate answers. The calculations performed with the other two methods generally took longer for most numbers. They also sometimes failed to reach the correct result. However, this new formula yielded both faster and more accurate results.

Conclusion and Discussion

In conclusion, we believe that our new formula offers a very practical and easy way to find the square of two-digit numbers. This allows for faster calculations without the need for pen and paper. We believe that this formula will also open new horizons for calculating the squares of three- or four-digit numbers, paving the way for practical formulas.