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Optimization of Educational Resources Allocation through fully fuzzy linear programming with Triangular Fuzzy Numbers

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Abstract

This study presents a Fully Fuzzy Linear Programming (FFLP) approach to optimize seat intake across multiple academic programs in an educational institute with the objective of maximizing profit. Triangular fuzzy numbers are employed to represent uncertainties in profits, costs, and resource constraints. To obtain actionable decisions, the fuzzy parameters are defuzzified into crisp values using a sub-interval ranking function. The resulting crisp model is solved through the Simplex Method, ensuring optimal allocation of limited resources such as faculty hours, classroom space, laboratory time, budget, and mentorship capacity. The methodology provides a systematic framework for decision makers to handle ambiguity while achieving profit maximization in academic seat planning.

Keywords: Fuzzy Linear Programming Problem, Ranking Function, Simplex Method, Triangular Fuzzy Numbers

1. Introduction

In today's competitive environment, making decisions that are timely, scientifically justified, and well reasoned plays a critical cole in determining the success or failure of organizations [1, 2). Because decision making processes often involve uncertainty and complexity, furzy theory has emerged as an effective approach for handling imprecision and achieving optimal decisions under such uncertain condidions [1. 4 The concept of luczy sets was first introduced by Lotfi A. Zadeh in 1965 [5]. Linear programming, a tundamental tool of operations research, has long been applied to solve a variety of optanization problems However, when parameters in fuzzy programming depend on expert judgment, they frequently cannot be defined with complete precision, as ambiguity and uncertainty are inevitable in human opinions. Fuzzy linear programming (FLP) addresses this issue by representing all decision variables and parameters in a fuzzy framework [6, 7]. To obtain optimal solutions for FLP model, several techniques have been proposed. Lotfi et al. (8) apprisached the problem by approximating parameters with symmetric triangular fuzzy numbers and then solving a multi-objective linear programming model to denve an approximate fuzzy optimal solution Kumar et al. [9] applied a linear ranking function to transform the fuzzy objective into a crisp form and derived a definite optimal result for fully fuzzy problems. Similarly, Ezzati et al. [10) introduced an algorithm that converts the problem into a multi objective linear program and solves it using a lexicographic method. This paper presents a new strategy for solving fully fuzzy linear programming (FFLP) problems by incorporating fuzzy decision variables and parameters represented with triangular fuzzy numbers. The method employs alpha-cut theory combined with modified triangular fuzzy numbers to obtain optimal solutions for practical applications, in this framework, the model is treated entirely as a fuzzy system parameters and variables are test expressed using the modified triangular fuzzy representation, after which the model is solved by considering the mid value as the objective function while using the upper and lower bounds as constraints. This procedure provides more precise optimal results and effectively captures the inherent uncertainty of the problem.

2. Prelimiaries:

2.1 Fuzzy Set: Let X be a non- empty set. A fuzzy set $\tilde{A}$ in X is charaterized by its membership function $\mu_{\tilde{A}}:X \rightarrow [0,1]$and $\mu_{\tilde{A}}(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$.

2.2Multi-Objective Fuzzy Linear Programming Problem:

If all the parameters of a linear programming Problem (LPP) are presented in terms of vagueness i.e fuzzy numbers then our LP problem is identified as fuzzy linear programming problem (FLPP) and if FLP problem consists of more than one objective for a particular modal then it's modal namely know as a multi-objective fuzzy linear programming problem. In ous study, fuzzy numbers assign as $\tilde{c}_r, \tilde{a}_{ij}$ etc. Here we consider MOFLPP as

$$Max\ or\ Min\ Z_s = \sum_{r=1}^{n} \tilde{c}_r^s x_r\ \forall\ s \in N$$

Subject to

$$\sum_{r=1}^{n} \tilde{a}_{ir} x_i \leq \tilde{b}_i \qquad 1 \leq i \leq m \ \exists\ x_i > 0$$

2.3 Triangular Fuzzy Number [TFN]:

A triangular fuzzy number is a type of fuzzy number which is used in decision making in uncertain environment. It represents by triplite (p, q, r) where:

P: least value of number.

q: modal value of number.

r: height value of number.

Notation $\tilde{\mathcal{A}}$ present the fuzzy number and defined by membership function $\mu_{\tilde{a}}(x)$. Value of membership function is belonged to 0 to 1.

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x-p}{q-p} & p \leq x \leq q \\ \dfrac{r-x}{r-q} & q \leq x \leq r \\ 0 & otherwise \end{cases}$$

2.4 Ranking Method for Triangular Fuzzy Numbers:

Ranking fuzzy numbers plays a vital role in solving problems, optimizing solutions, and making decisions under uncertainty. Since fuzzy numbers represent vague or imprecise information, their comparison and prioritization are critical in areas such as artificial intelligence, economics, and engineering. Different types of fuzzy numbers-like triangular, trapezoidal, and Gaussian-require specific ranking methods to ensure accuracy and consistency. Common approaches include α-cut methods, distance-based measures, and centroid calculations. An effective ranking method reduces ambiguity, enhances computational efficiency. and enables reliable evaluation of alternatives. Accurate ranking is therefore essential for real world applications such as risk assessment, supplier selection, and multi-criteria decision-making in uncertain environments.

Sub Interval Average[11]: By dividing a triangle fuzzy number into smaller intervals and averaging their midpoints for defuzzification, the Sub Interval Average technique improves decision-making accuracy by capturing interval variability. The formula $A = (a_1, a_2, a_3)$ is stated as follows

$$\boldsymbol{R}(p,q,r) = \frac{\mathbf{4}(p+,q+r)}{\mathbf{12}} \qquad (2.1)$$

2.5 Simplex Method:

Step 1:Formulate the Problem

a. Formulate the mathematical model of the given linear programming problem.

b. If the objective function is minimization type, then change it into maximization type

$$Min\ z = -Max\ (-z)$$

c. All the $X_{B_i} > 0$. So, if any$X_{B_i} < 0$ then multiply the corresponding constraint by -1 to make $X_{B_i} > 0$. So sign $\leq$ changed to $\geq$ and vice varsa.

d. Transform every $\leq$ constraint into an $=$ constraint by adding a slack variable to every constraint and assign a 0 cost coefficient in the objective function.

Step 2:Find out the Initial basic solution

Find the initial basic feasible solution by setting zero value to the decision variables.

Step 3: Test for Optimality

a. Calculate the values of $Z_j - C_j$ in the last row of simplex table.

b. If all $Z_j - C_j > 0$, the current basic feasible solution is the optimal solution.

c. In $Z_j - C_j < 0$, then select the variable that has largest $Z_j - C_j$ and enter this variable into the new table. This column is called key column (pivot column).

Step 4:Test for Feasibility (variable to leave the basis)

a. Find the ratio by dividing the values of X_B column by the positive values of key column (say $a_{ij} > 0$)

b. Find the minimum ratio and this row is called key row (pivot row) and corresponding variable will leave the solution.

c. The intersection element of key row and key column is called key element (pivot element).

Step 5:Determine the new solution

a. The new values of key row can be obtained by dividing the key row elements by the pivot element.

b. The numbers in the remaining rows can be computed by utilizing the following formula:
Row(new) = Row(old) - (value of key column and Row(old)) × KeyRow(new)

Step 6:Repat the procedure.

Goto step 3 and repeat the procedure until all the values of $Z_j - C_j \geq 0$.

3. Numerical Example: An Institute plans how many seats to open this year in 6 programmes to maximize fuzzy annual revenue. Many inputs are uncertain (revenues, faculty, hours needed per seat, budget available, etc)-so all coefficients and rights-hand sides are modelled as triangular fuzzy numbers.

Let the decision variables be:

x_1= B Tech seats | x_2= MBA seats | x_3= M. Sc. seats

x_4= B. Ed. Seats | x_5= Nursing seats | x_6= Diploma seats

Objective

$$\tilde{Z} = (1199, 1209,1219)x_1 + (1281,1291,1301)x_2 + (919,929,939)x_3 + (480,500,520)x_4 + (1069,1079,1089)x_5 + (290,300,310)x_6$$

Subject to Constraints

Faculty hours availability (hours faculty spend per seat)

$$(2.9,3.0,3.1)x_1 + (3.9,4.0,4.1)x_2 + (1.9,2.0,2.1)x_3 + (1.9,2.0,2.1)x_4 + (2.9,3.0,3.1)x_5 + (0.1,1.0,1.1)x_6 \leq (920,940,960) \text{ (hous/years)}$$

Classroom hours availability

$$(4.9,5.0, 5.1)x_1 + (5.9,6.0,6.1)x_2 + (3.9,4.0,4.1)x_3 + (2.9,3.0,3.1)x_4 + (3.9,4.0,4.1)x_5 + (1.9,2.0,2.1)x_6 \leq (1420,1440,1460) \text{ (hous/years)}$$

Laboratory hours availability

$$(5.5,6.0,6.5)x_1 + (1.5,2.0,2.5)x_2 + (4.5,5.0,5.5)x_3 + (0.5,1.0,1.5)x_4 + (3.5,4.0,4.5)x_5 + (0.5,1.0,1.5)x_6 \leq (1310,1330,1350) \text{ (hous/years)}$$

Budget for new seat setup

$$(19.5,20.0,20.5)x_1 + (29.5,30.0,30.5)x_2 + (14.5,15.0,15.5)x_3 + (9.5,10,10.5)x_4 + (24.5,25.,25.5)x_5 + (7.5,8.0,8.5)x_6 \leq (7100,7150,7200) \text{ (thousands)}$$

Total seat capacity

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq (300,310,320) \text{ (seats)}$$

Mentorship (Mentoring load per seat)

$$(1.5,2.0,2.5)x_1 + (2.9,3.0,3.1)x_2 + (1.5,2.0,2.5)x_3 + (0.9,1.2,1.1)x_4 + (1.5,2.0,2.5)x_5 + (0.9,1.2,1.1)x_6 \leq (660,680,700)$$

After the implementation of Sub interval ranking method our triangular fuzzy numbers change into crisp number and fuzzy linear programming problem convert into linear programming problem as follows

Objective

$$Z = 1209x_1 + 1291x_2 + 929x_3 + 550x_4 + 1079x_5 + 300x_6$$

Subject to Constraints

Faculty hours availability

$$4x_1 + 3x_2 + 2x_3 + 2x_4 + 3x_5 + 1x_6 \leq 940$$

Classroom hours availability

$$5x_1 + 6x_2 + 4x_3 + 3x_4 + 4x_5 + 2x_6 \leq 1440$$

Laboratory hours availability

$$6x_1 + 2x_2 + 5x_3 + 1x_4 + 4x_5 + 1x_6 \leq 1330$$

Budget for new seat setup

$$20x_1 + 30x_2 + 15x_3 + 10x_4 + 25x_5 + 8x_6 \leq 7150$$

Total seat capacity

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 310$$

Mentorship

$$2x_1 + 3x_2 + 2x_3 + 1x_4 + 2x_5 + 1x_6 \leq 680$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

After introducing slack variables

Objective

$$Z = 1209x_1 + 1291x_2 + 929x_3 + 550x_4 + 1079x_5 + 300x_6 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 + 0s_6$$

Subject to Constraints

$$4x_1 + 3x_2 + 2x_3 + 2x_4 + 3x_5 + 1x_6 + 0s_1 = 940$$

$$5x_1 + 6x_2 + 4x_3 + 3x_4 + 4x_5 + 2x_6 + 0s_2 = 1440$$

$$6x_1 + 2x_2 + 5x_3 + 1x_4 + 4x_5 + 1x_6 + 0s_3 = 1330$$

$$20x_1 + 30x_2 + 15x_3 + 10x_4 + 25x_5 + 8x_6 + 0s_4 = 7150$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + +0s_5 = 310$$

$$2x_1 + 3x_2 + 2x_3 + 1x_4 + 2x_5 + 1x_6 + 0s_6 = 680$$

$$x_1, x_2, x_3, x_4, x_5, x_6, s_1, s_2, s_3, s_4, s_5, s_6 \geq 0$$

Table 1

		C_j	1209	1291	929	500	1079	300	0	0	0	0	0	0	X_B/x_2
B	C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6	s_1	s_2	s_3	s_4	s_5	s_6	
s_1	0	940	3	4	2	2	3	1	1	0	0	0	0	0	235
s_2	0	1440	5	6	4	3	4	2	0	1	0	0	0	0	240
s_3	0	1330	6	2	5	1	4	1	0	0	1	0	0	0	665
s_4	0	7150	20	30	15	10	25	8	0	0	0	1	0	0	238.33
s_5	0	310	1	1	1	1	1	1	0	0	0	0	1	0	310
s_6	0	680	2	3	2	1	2	1	0	0	0	0	0	1	226.67
		Z_j	0	0	0	0	0	0	0	0	0	0	0	0	
	$Z_j - C_j$		-1209	-1291	-929	-500	-1079	-300	0	0	0	0	0	0	

Negative minimum $Z_j - C_j$ is -1291 and its column index is 2. So entering variable is x_2. Minimum ratio is 226.67 and its row index is 6. So the leaving basic variable is s_6.

Table 2

		C_j	1209	1291	929	500	1079	300	0	0	0	0	0	0	X_B/x_2
B	C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6	s_1	s_2	s_3	s_4	s_5	s_6	
s_1	0	33.33	0.33	0	-0.67	0.67	0.33	-0.33	1	0	0	0	0	-1.33	100
s_2	0	80	1	0	0	1	0	0	0	1	0	0	0	-2	80
s_3	0	876.67	4.67	0	3.67	0.33	2.67	0.33	0	0	1	0	0	-0.67	187.86
s_4	0	350	0	0	-5	0	5	-2	0	0	0	1	0	0	--
s_5	0	83.33	0.33	0	0.33	0.67	0.33	0.67	0	0	0	0	1	-0.33	250
s_6	0	226.97	0.67	1	0.67	0.33	0.67	0.33	0	0	0	0	0	0.33	340
		Z_j	860.67	1291	860.67	430.33	860.67	430.33	0	0	0	0	0	430.33	

	$Z_j - C_j$	-348.33	0	-68.33	-69.67	-218.33	130.33	0	0	0	0	0	0	

Now we are going to apply simplex method to get the optimal solution, here we found the optimal solution as follows

Table 3

		C_j	1209	1291	929	500	1079	300	0	0	0	0	0	0
B	C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6	s_1	s_2	s_3	s_4	s_5	s_6
x_5	1079	120	0	0	0	1	1	1	1	-1	0	0	2	0
x_1	1209	80	1	0	0	-1.8	0	-2.4	0.4	0.2	0.4	0	-3.6	0
s_3	0	0	0	0	0	-1.4	0	-1.2	0.2	-0.4	0.2	0	-1.8	1
s_4	0	0	0	0	0	-12	5	-8	3	1	1	1	-14	0
x_3	929	50	0	0	1	1.4	0	2.2	-1.2	0.4	-0.2	0	2.8	0
x_2	1291	60	0	1	0	0.4	0	0.2	-0.2	0.4	-0.2	0	-0.2	0
		Z_j	1209	1291	929	719.8	1079	479.4	189.6	50.8	39.6	0	148.6	0
	$Z_j - C_j$		0	0	0	219.8	0	179.4	189.4	50.8	39.6	0	148.6	0

Since all $Z_j - C_j \geq 0$

Hence the optimal solution is arrived with the values of variables as:

Table 4

S No	Name of Course	Assign Variable	No of Seats
1	B Tech	x_1	80
2	MBA	x_2	60
3	M. Sc.	x_3	50
4	B Ed	x_4	0
5	Nursing	x_5	120
6	Diploma	x_6	0

Put the above values in define objective function to get the maximum profit

$$Z = 1209 \times 80 + 1291 \times 60 + 929 \times 50 + 550 \times 0 + 1079 \times 120 + 300 \times 0$$

$$Z = 350110$$

4. Conclusion:

The proposed Fully Fuzzy Linear Programming model successfully demonstrated how triangular fuzzy numbers can effectively manage uncertainties in educational resource planning. By converting fuzzy values into crisp equivalents using the sub-interval ranking approach, the study provided a reliable basis for applying the Simplex Method. The optimized solution indicated that the institute should allocate 80 seats in B Tech, 60 in MBA, 50 in M.Sc, and 120 in Nursing. while avoiding expansions in B.Ed and Diploma. This allocation fully utilized faculty hours, classroom availability, laboratory resources, budget, and mentorship capacity, thereby maximizing institutional profit. The results validate the potential of fuzzy-based optimization as a strategic tool in academic planning. Future research can extend this model to incorporate dynamic constraints and multi-objective criteria, further enhancing decision-making robustness in educational institutions.

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