

Vertex Antimagic Edge Slither Labeling of Diamond Ladder Graph

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Abstract

The interconnected systems are one of the most advanced applications science and technology that enables fast and hassle-free transfer of data. As they evolve over time, it is important to important to take these applications to every house-hold of the common man. To facilitate the upgradation of interconnected networks, it is important to have a tool that is instrumental in simplifying these complex structures. One such tool is the diamond ladder graph. In this paper, the applicability of vertex antimagic edge slither labeling of the diamond ladder graph is established.

Keywords: Diamond Ladder Graph, Antimagic Labeling, Slither Labeling.

1. Introduction

The interconnected networks has gamut of applications that has brought the world within the reach of the common man. To make this possible, the diamond ladder graph plays a very vital role. The concept of labeling is one of the most non-enigmatic topics in graph theory. There are a wide a range of labeling types that has applications far and wide in all spheres of the universe. Out of the different types of labeling, the one that provided the impetus is the concept of antimagic labeling. The labeling is the assignment of positive integers to the vertices or edges or both. The idea of antimagic labeling was first proposed by Martin Baca and Mirka Miller [4] who defined antimagic labeling in graphs as follows; A graph G is said to possess (a, d) antimagic labeling if there exists a positive integer a and a non-negative integer d and a bijection $h_1: E \rightarrow \{1, 2, \dots, |E(G)|\}$ such that the induced mapping given by the function $h_1^*: V(G) \rightarrow W$, where $W = \{a, a+d, a+2d, \dots, a+(|V|-1)/d\}$ is also a bijection. J. Sedlacek [7] was the first person to give the notion of magic labeling. This concept was further extended to vertex magic labeling by J.A. Macdougall, M. Miller and W.D. Wallis [5] [6]. The idea of antimagic labeling was first given by Hartsfield and Ringel [3]. The concept of (a, d) antimagic labeling in graphs was first proposed by the eminent graph labeling researchers R. Bodendiek and G. Walther [1]. The notion of vertex antimagic edge slither labeling is a variation of vertex antimagic edge labeling. All these were classified and compiled by J.A. Gallian [2].

Diamond Ladder Graph: A diamond graph is a graph formed from K_4 by removing a diagonal edge. A diamond ladder graph, denoted by 'DL_n' is a graph that is obtained by joining numerous diamond graph in a path on n vertices, that is similar to a ladder formation. The number of vertices in the graph is $4n$.

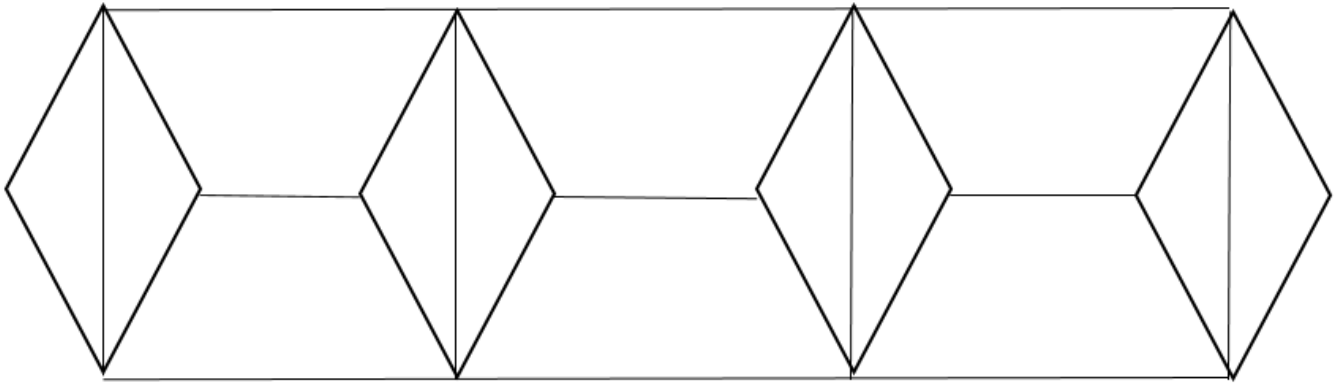


Fig 1. Diamond Ladder Graph DL_4

Main Result:

Theorem: The diamond Ladder graph admits vertex antimagic edge slither labeling for $n \geq 2$.

Proof: Consider the diamond ladder graph DL_n , ($n \geq 2$). Label the edges of the graph in the slither pattern. The label of a vertex is the sum of labels of the edges that are incident with that vertex.

Case – (1): Consider the graph DL_3 . The slither labeling of the edges are given as follows;

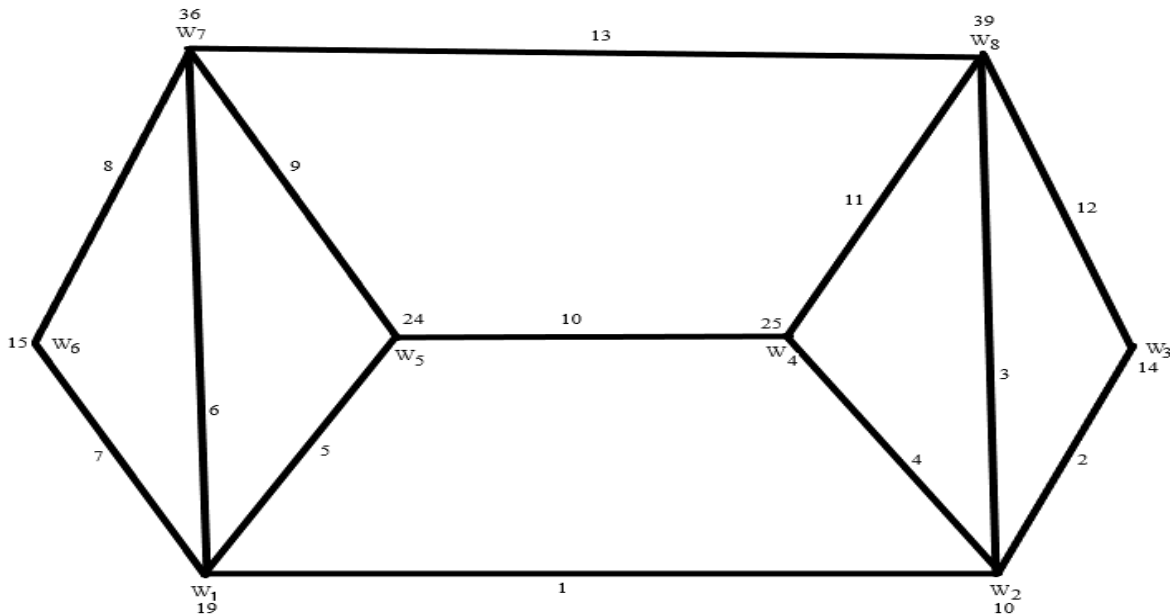


Fig 1. Diamond Ladder Graph DL_2

The edges of the graph are distinctively following the slither pattern and the labels of the vertices are also distinct. Hence the graph admits vertex antimagic edge slither labeling.

Case – (2): Consider the graph DL_3 . The slither labeling of the edges are given as follows;

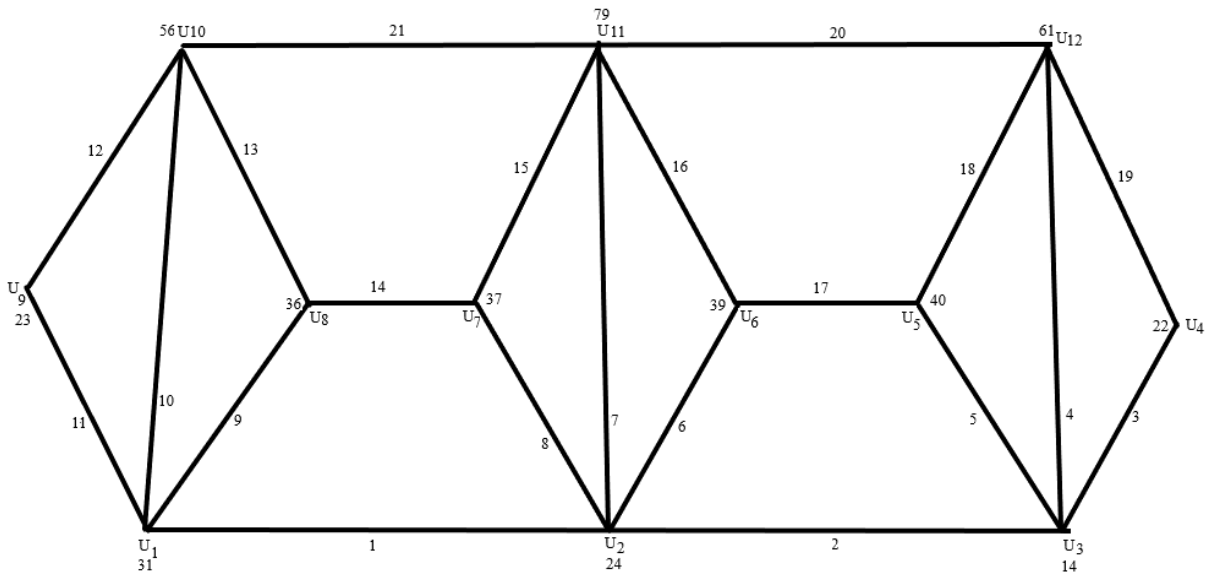


Fig 3. Diamond Ladder Graph DL_3

The edges of the graph are distinctively following the slither pattern and the labels of the vertices are also distinct. Hence the graph admits vertex antimagic edge slither labeling.

Case – (3): Consider the graph DL_4 . The slither labeling of the edges are given as follows;

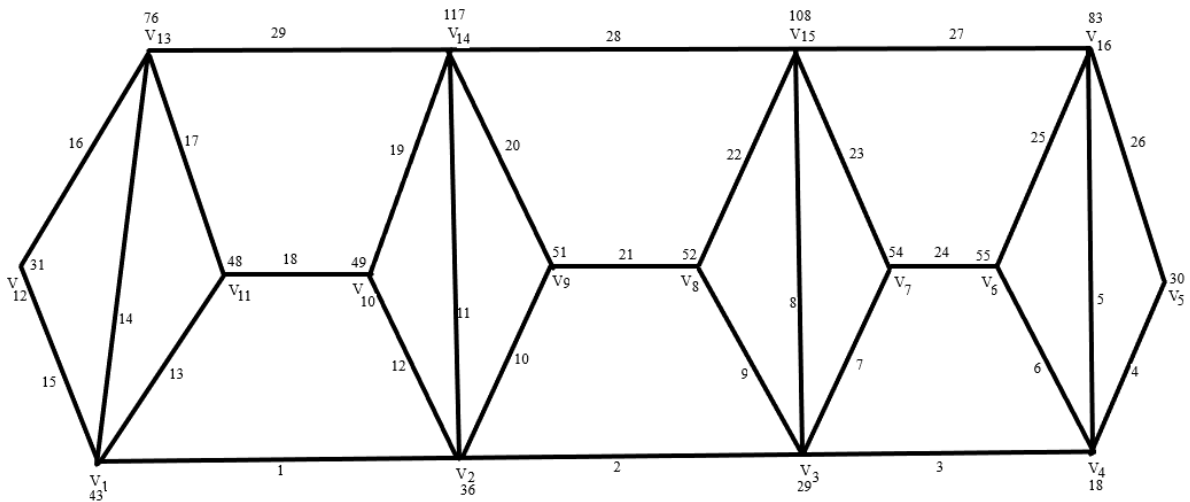


Fig 4. Diamond Ladder Graph DL_4

The edges of the graph are distinctively following the slither pattern and the labels of the vertices are also distinct. Hence the graph admits vertex antimagic edge slither labeling.

This process of slither labeling holds true for all values of $n \geq 2$. Hence the graph DL_n admits vertex antimagic edge slither labeling.

Conclusion

In this paper, the diamond ladder graph is considered and it has been proved that the graph admits vertex antimagic edge slither labeling. In a similar way, different graphs can be taken and the applicability of vertex antimagic edge slither labeling can be attempted.

References

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