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Graph Theorem Approach to ω - Pentagonal Fuzzy Bottleneck Assignment Problems

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Abstract

In this paper we presented a spread of minimum solution of fuzzy optimization matching procedure in the bipartite graph. It provides minimum vertex cover with edge set E for solving ω Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problems (ω - PFLSAP). The ω pentagonal fuzzy linear sum bottleneck assignment problem is minimum cost and maximum matching in the bipartite graph. The Linear Sum Bottleneck Assignment Cost (LSBAC) we taken as ω Pentagonal Fuzzy Numbers (ω-PFN). Fuzzy figuring essential parts in issues in the dynamic, examination of information, and economics course of action. Tracking down the Positioning of any Fuzzy numbers is an unavoidable advance in numerous numerical models. A Significant number of the strategies proposed, which created the best answer for the transportation issues. This paper presents a proposed positioning technique applying a similar we change the generalized Fuzzy transportation issue to a dazzling esteemed one, hence into another proposed interaction to uncover the Fuzzy reasonable arrangement. The mathematical representation exhibits that the new projected technique delicate marvellous methods for dealing with the transportation issues on Fuzzy calculation.

Keywords: Fuzzy Number, ω - Pentagonal Fuzzy numbers, ω Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problems, Linear Sum Bottleneck Assignment Cost.

1. Introduction

In 1987, Zadeh [14] introduced fuzzy set as a basis for a theory f possibility. In 1996, Channas and Kuchta [15] discussed a concept of optimal solution of the the transportation problem with fuzzy cost coefficient. Kuhn [6] presented the Hungarian method for the assignment problem in the year of 2005. In 2010, Mukherjee and Basu [7] discussed application of fuzzy ranking method for solving assignment problem with fuzzy costsand De and Yadav [8] presented a general approach for solving assignment problems involving with fuzzy cost coefficients. In 2015, Panda and Pal [13] exposed a study on pentagonal fuzzy number and is corresponding matrices and Jatinder Pal Singh and Neha Ishesh Thakur [20] exposed a novel method to solve assignment problem in fuzzy environment. AnchalChoudhary et al presented a new algorithm for solving fuzzy assignment problem using branch and bound methold. In the year of 2017 AvinashKamble [21] discussed some notes o pentagonal fuzzy numbers.

In this paper, we discussed minimum vertex cover of perfect matching solution (μ) for solving optimal solution of ω - pentagonal fuzzy linear sum bottleneck assignment problem.

2. Preliminaries

In this chapter discussed basic concepts of fuzzy sets and ω- trapezoidal fuzzy numbers, and ω -pentagonal fuzzy numbers, spread of solution.

2.1 Definition.

A fuzzy set is defined as a pair of (X, $\mu(x)$) in which X is a set and $\mu(x)$ = X$\rightarrow$[0,1] is a membership function. For each x $\in$X, the set X is referred to as the universe of discourse. The value $\mu(x)$ is called the grade of membership of x in (X, $\mu(x)$). The function $\mu(x)$ is called the membership function of the fuzzy set $\overline{P}$ = (X, $\mu(x)$).

2.2 Definition.

A fuzzy number $\overline{P}$ = (p_1, p_2, p_3, $p_4\omega$) is said to be a ω trapezoidal fuzzy number if its membership function

$$\mu_{\overline{P}}(x)=\begin{cases}\omega\left(\dfrac{x-p_1}{p_2-p_1}\right) & for \quad p_1\le x\le p_2\\ \omega & for \quad p_2\le x\le p_3 \quad where\, \omega\in(0,1)\\ \omega\left(\dfrac{p_4-x}{p_4-p_3}\right) & for \quad p_3\le x\le p_4\end{cases}$$

2.3 Definition.

ωP.F.N of a fuzzy set $\overline{P}$ is denoted as $\overline{P}$ = (p_1, p_2, p_3, p_4, p_5, ω) where P_3 is the middle point and (P_1, P_2) and (P_4, P_5) are the left and right side points of P_3, respectively and its membership function is given by

$$\mu_{\overline{P}}(x)=\begin{cases}\dfrac{1}{2}\left(\dfrac{x-p_1}{p_2-p_1}\right) & for \quad p_1\le x\le p_2\\ \dfrac{1}{2}\left(\dfrac{1}{2}-\omega\right)\left(\dfrac{x-p_2}{p_3-p_2}\right) & for \quad p_2\le x\le p_3\\ \omega & for \quad \mathrm{x}=\mathrm{p}_3 \quad where\, \omega\in(0.5,1)\\ \dfrac{1}{2}-\left(\dfrac{1}{2}-\omega\right)\left(\dfrac{p_4-x}{p_4-p_3}\right) & for \quad p_3\le x\le p_4\\ \dfrac{1}{2}\left(\dfrac{p_5-x}{p_5-p_4}\right) & for \quad p_4\le x\le p_5\end{cases}$$

3. Ranking of ω - Pentagonal Fuzzy numbers

In this section, we discuss about the numerical example of ω-Pentagonal Fuzzy Number using Ranking function.

Let A be a ω - Pentagonal Fuzzy Number. The Ranking of A ie., R(A)is calculated as follows:

$$R(A) = \frac{a_1 + 2a_2 + 2a_3 + 2a_4 + a_5}{8}$$

The table is evaluating each and every assignment permutation to determine the most effective one by utilizing,

- Pentagonal Fuzzy numbers
- Ranking Method
- Count of Borda (number of ranks)
- Limit (the highest rank)

The combination that includes

- Minimum Borda sum
- Minimum bottleneck value

Final Observation

- It generates every possible assignment permutation.
- The corresponding pentagonal fuzzy numbers are chosen and ranked for each permutation.
- By adding up the values of the ranking, the borda count is calculated.
- The bottleneck value is taken as the maximum rank.
- The best assignment is the permutation with the lowest Borda sum and lowest bottleneck value.

Numerical Example:

Consider the following cost of minimization assignment problem.

Table 1

	J_1	J_2	J_3	J_4
P_1	(1,2,3,4,5)	(2,3,4,5,6)	(3,4,5,6,7)	(4,5,6,7,8)
P_2	(2,3,4,5,6)	(3,4,5,6,7)	(4,5,6,7,8)	(1,2,3,4,5)
P_3	(3,4,5,6,7)	(4,5,6,7,8)	(1,2,3,4,5)	(2,3,4,5,6)
P_4	(4,5,6,7,8)	(1,2,3,4,5)	(2,3,4,5,6)	(3,4,5,6,7)

Objective minimize total fuzzy cost (or use Bottleneck) using Permutation method.

Convert P.F.N to scalar, for comparison method.

Using ranking method

$R_{i,j}(A) = \dfrac{a_1 + 2a_2 + 2a_3 + 2a_4 + a_5}{8}$ for i = 1 to 4 and j = 1 to 4

R_{11} (or) R(1, 2, 3, 4, 5) = 3, $R_{12} = 4$, $R_{13} = 5$, $R_{14} = 6$

Converted crisp Matrix

$$\begin{pmatrix} 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 3 \\ 5 & 6 & 3 & 4 \\ 6 & 3 & 4 & 5 \end{pmatrix}$$

List of all Bipartite Graph Representation as follows:

i. $P_1 \rightarrow J_1 = 3$, $P_1 \rightarrow J_2 = 4$, $P_1 \rightarrow J_3 = 5$, $P_1 \rightarrow J_4 = 6$

ii. $P_2 \rightarrow J_1 = 4$, $P_2 \rightarrow J_2 = 5$, $P_2 \rightarrow J_3 = 6$, $P_2 \rightarrow J_4 = 3$

iii. $P_3 \rightarrow J_1 = 5$, $P_3 \rightarrow J_2 = 6$, $P_3 \rightarrow J_3 = 3$, $P_3 \rightarrow J_4 = 4$

iv. $P_4 \rightarrow J_1 = 6$, $P_4 \rightarrow J_2 = 3$, $P_4 \rightarrow J_3 = 4$, $P_4 \rightarrow J_4 = 5$

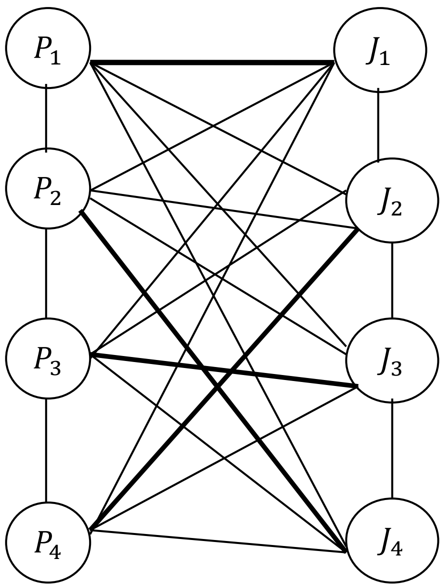

figure 1

List of all permutations

1. $P_1 \rightarrow J_1$, $P_1 \rightarrow J_2$, $P_1 \rightarrow J_3$, $P_1 \rightarrow J_4$
2. $P_1 \rightarrow J_1$, $P_1 \rightarrow J_2$, $P_1 \rightarrow J_4$, $P_1 \rightarrow J_3$
3. $P_1 \rightarrow J_1$, $P_1 \rightarrow J_3$, $P_1 \rightarrow J_2$, $P_1 \rightarrow J_4$
4. $P_1 \rightarrow J_1$, $P_1 \rightarrow J_3$, $P_1 \rightarrow J_4$, $P_1 \rightarrow J_2$
5. $P_1 \rightarrow J_1$, $P_1 \rightarrow J_4$, $P_1 \rightarrow J_2$, $P_1 \rightarrow J_3$
6. $P_1 \rightarrow J_1$, $P_1 \rightarrow J_4$, $P_1 \rightarrow J_3$, $P_1 \rightarrow J_2$
7. $P_2 \rightarrow J_2$, $P_2 \rightarrow J_1$, $P_2 \rightarrow J_3$, $P_2 \rightarrow J_4$
8. $P_2 \rightarrow J_2$, $P_2 \rightarrow J_1$, $P_2 \rightarrow J_4$, $P_2 \rightarrow J_3$
9. $P_2 \rightarrow J_2$, $P_2 \rightarrow J_3$, $P_2 \rightarrow J_1$, $P_2 \rightarrow J_4$
10. $P_2 \rightarrow J_2$, $P_2 \rightarrow J_3$, $P_2 \rightarrow J_4$, $P_2 \rightarrow J_1$
11. $P_2 \rightarrow J_2$, $P_2 \rightarrow J_4$, $P_2 \rightarrow J_1$, $P_2 \rightarrow J_3$
12. $P_2 \rightarrow J_2$, $P_2 \rightarrow J_4$, $P_2 \rightarrow J_3$, $P_2 \rightarrow J_1$
13. $P_3 \rightarrow J_3$, $P_3 \rightarrow J_1$, $P_3 \rightarrow J_2$, $P_3 \rightarrow J_4$
14. $P_3 \rightarrow J_3$, $P_3 \rightarrow J_1$, $P_3 \rightarrow J_4$, $P_3 \rightarrow J_2$
15. $P_3 \rightarrow J_3$, $P_3 \rightarrow J_2$, $P_3 \rightarrow J_1$, $P_3 \rightarrow J_4$
16. $P_3 \rightarrow J_3$, $P_3 \rightarrow J_2$, $P_3 \rightarrow J_4$, $P_3 \rightarrow J_1$
17. $P_3 \rightarrow J_3$, $P_3 \rightarrow J_4$, $P_3 \rightarrow J_1$, $P_3 \rightarrow J_2$
18. $P_3 \rightarrow J_3$, $P_3 \rightarrow J_4$, $P_3 \rightarrow J_2$, $P_3 \rightarrow J_1$
19. $P_4 \rightarrow J_4$, $P_4 \rightarrow J_1$, $P_4 \rightarrow J_2$, $P_4 \rightarrow J_3$
20. $P_4 \rightarrow J_4$, $P_4 \rightarrow J_1$, $P_4 \rightarrow J_3$, $P_4 \rightarrow J_2$
21. $P_4 \rightarrow J_4$, $P_4 \rightarrow J_2$, $P_4 \rightarrow J_1$, $P_4 \rightarrow J_3$
22. $P_4 \rightarrow J_4$, $P_4 \rightarrow J_2$, $P_4 \rightarrow J_3$, $P_4 \rightarrow J_1$
23. $P_4 \rightarrow J_4$, $P_4 \rightarrow J_3$, $P_4 \rightarrow J_1$, $P_4 \rightarrow J_2$
24. $P_4 \rightarrow J_4$, $P_4 \rightarrow J_3$, $P_4 \rightarrow J_2$, $P_4 \rightarrow J_1$

S. No	PermutationAssignment (P_1, P_2, P_3, P_4)	SelectedP.F.N	Ranking**V**alues	Sum	Max (Bottleneck)
1.	$P_1 \rightarrow J_1$, $P_2 \rightarrow J_2$ $P_3 \rightarrow J_3$, $P_4 \rightarrow J_4$	(1,2,3,4,5) (3,4,5,6,7) (1,2,3,4,5) (3,4,5,6,7)	(3, 5, 3, 5)	16	5
2.	$P_1 \rightarrow J_1$, $P_2 \rightarrow J_2$ $P_3 \rightarrow J_4$, $P_4 \rightarrow J_3$	(1,2,3,4,5) (3,4,5,6,7) (2,3,4,5,6) (2,3,4,5,6)	(3, 5, 4, 4)	16	5
	$P_1 \rightarrow J_1$, $P_2 \rightarrow J_3$ $P_3 \rightarrow J_2$, $P_4 \rightarrow J_4$	(1,2,3,4,5) (4,5,6,7,8) (4,5,6,7,8) (3,4,5,6,7)	(3, 6, 6, 5)	20	6
4.	$P_1 \rightarrow J_1$, $P_2 \rightarrow J_3$ $P_3 \rightarrow J_4$, $P_4 \rightarrow J_2$	(1,2,3,4,5) (4,5,6,7,8) (2,3,4,5,6)(1,2,3,4,5)	(3, 6, 4, 3)	16	6
5.	$P_1 \rightarrow J_1$, $P_2 \rightarrow J_4$ $P_3 \rightarrow J_2$, $P_4 \rightarrow J_3$	(1,2,3,4,5)(1,2,3,4,5) (4,5,6,7,8) (2,3,4,5,6)	(3, 3, 6, 4)	16	6
6.	$P_1 \rightarrow J_1$, $P_2 \rightarrow J_4$ $P_3 \rightarrow J_3$, $P_4 \rightarrow J_2$	(1,2,3,4,5)(1,2,3,4,5) (1,2,3,4,5)(1,2,3,4,5)	(3, 3, 3, 3)	**12**	**3**
7.	$P_1 \rightarrow J_2$, $P_2 \rightarrow J_1$	(2,3,4,5,6) (2,3,4,5,6)	(4, 4, 3, 5)	16	5

	$P_3 \to J_3, P_4 \to J_4$	(1,2,3,4,5) (3,4,5,6,7)			
8.	$P_1 \to J_2, P_2 \to J_1$ $P_3 \to J_4, P_4 \to J_3$	(2,3,4,5,6) (2,3,4,5,6) (2,3,4,5,6) (2,3,4,5,6)	(4, 4, 4, 4)	16	4
9	$P_1 \to J_2, P_2 \to J_3$ $P_3 \to J_1, P_4 \to J_4$	(2,3,4,5,6) (4,5,6,7,8) (3,4,5,6,7) (3,4,5,6,7)	(4, 6, 5, 5)	20	6
10.	$P_1 \to J_2, P_2 \to J_3$ $P_3 \to J_4, P_4 \to J_1$	(2,3,4,5,6) (4,5,6,7,8) (2,3,4,5,6) (4,5,6,7,8)	(4, 6, 4, 6)	20	6
11.	$P_1 \to J_2, P_2 \to J_4$ $P_3 \to J_1, P_4 \to J_3$	(2,3,4,5,6)(1,2,3,4,5) (3,4,5,6,7) (2,3,4,5,6)	(4, 3, 5, 4)	16	5
12.	$P_1 \to J_2, P_2 \to J_4$ $P_3 \to J_3, P_4 \to J_1$	(2,3,4,5,6)(1,2,3,4,5) (1,2,3,4,5) (4,5,6,7,8)	(4, 3, 3, 6)	16	6
13.	$P_1 \to J_3, P_2 \to J_1$ $P_3 \to J_2, P_4 \to J_4$	(3,4,5,6,7)(2,3,4,5,6) (4,5,6,7,8) (3,4,5,6,7)	(5, 4, 6, 5)	20	6
14.	$P_1 \to J_3, P_2 \to J_1$ $P_3 \to J_4, P_4 \to J_2$	(3,4,5,6,7) (2,3,4,5,6) (2,3,4,5,6)(1,2,3,4,5)	(5, 4, 4, 3)	16	5
15.	$P_1 \to J_3, P_2 \to J_2$ $P_3 \to J_1, P_4 \to J_4$	(3,4,5,6,7) (3,4,5,6,7) (3,4,5,6,7) (3,4,5,6,7)	(5, 5, 5, 5)	20	5
16.	$P_1 \to J_3, P_2 \to J_2$ $P_3 \to J_4, P_4 \to J_1$	(3,4,5,6,7) (3,4,5,6,7) (2,3,4,5,6) (4,5,6,7,8)	(5, 5, 4, 6)	20	6
17.	$P_1 \to J_3, P_2 \to J_4$ $P_3 \to J_1, P_4 \to J_2$	(3,4,5,6,7)(1,2,3,4,5) (3,4,5,6,7)(1,2,3,4,5)	(5, 3, 5, 3)	16	5
18.	$P_1 \to J_3, P_2 \to J_4$ $P_3 \to J_2, P_4 \to J_1$	(3,4,5,6,7)(1,2,3,4,5) (4,5,6,7,8) (4,5,6,7,8)	(5, 3, 6, 6)	20	6
19.	$P_1 \to J_4, P_2 \to J_1$ $P_3 \to J_2, P_4 \to J_3$	(4,5,6,7,8) (2,3,4,5,6) (4,5,6,7,8) (2,3,4,5,6	(6, 4, 6, 4)	20	6
20.	$P_1 \to J_4, P_2 \to J_1$ $P_3 \to J_3, P_4 \to J_2$	(4,5,6,7,8) (2,3,4,5,6) (1,2,3,4,5)(1,2,3,4,5)	(6, 4, 3, 3)	16	6
21.	$P_1 \to J_4, P_2 \to J_2$ $P_3 \to J_1, P_4 \to J_3$	(4,5,6,7,8)(3,4,5,6,7) (3,4,5,6,7) (2,3,4,5,6)	(6, 5, 5, 4)	20	6
22.	$P_1 \to J_4, P_2 \to J_2$ $P_3 \to J_3, P_4 \to J_1$	(4,5,6,7,8)(3,4,5,6,7) (1,2,3,4,5) (4,5,6,7,8)	(6, 5, 3, 6)	20	6
23.	$P_1 \to J_4, P_2 \to J_3$ $P_3 \to J_1, P_4 \to J_2$	(4,5,6,7,8)(4,5,6,7,8) (3,4,5,6,7) (1,2,3,4,5)	(6, 6, 5, 3)	20	6
24.	$P_1 \to J_4, P_2 \to J_3$ $P_3 \to J_2, P_4 \to J_1$	(4,5,6,7,8)(4,5,6,7,8) (4,5,6,7,8)(4,5,6,7,8)	(6, 6, 6, 6)	24	6

Table 1

RESULT

1. Optimal sum assignment = Permutation (6)

   ($P_1 \to J_1$, $P_2 \to J_4$, $P_3 \to J_3$, $P_4 \to J_2$)

   = 12

2. Optimal Bottleneck Assignment = Permutation (6)

   ($P_1 \to J_1$, $P_2 \to J_4$, $P_3 \to J_3$, $P_4 \to J_2$)

   = maximum bottleneck

   = 3

References

1. Abinav Bansal, Trapezoidal Fuzzy Numbers (a, b, c, d): Arithmetic behavior, International Journal of Physical and Mathematical Sciences, ISSN – 2010 – 1791.
2. Apurba Panda, Madhumangal Pal, A study on pentagonal fuzzy number and its corresponding matrices pp 131 – 139, 2015.
3. Avinash J.Kamble, Some notes on Pentagonal Fuzzy Numbers, International Journal of Mathematical Archive, vol 13, no. 2, pp 113 – 121, 2017.
4. Bortolan G, Degani R, A review of some methods for ranking fuzzy subsets, Fuzzy Set and Systems, vol. 15, pp 1 – 19, 1985.
5. Chen SJ, Hwang CL, Fuzzy multiple attribute decision making, Springer, 1992.
6. Chu T,Tsao C, Ranking of Fuzzy Numbers with an Area between the Centroid and Original Points,Computers and Mathematics with application, vol 43, pp 111-117, 2002.
7. Jatinder Pal Singh Neha Ishesh Thakur, A Novel Method to Solve Assignment Problem in Fuzzy Environment, pp 2224-6096, pp 2225-0581 Vol.5, No.2, 2015.
8. H. W. Kuhn, 2005. " The Hungarian method for the assignment problem," Naval Research Logistics (NRL), John Wiley & Sons, vol. 52 (1), pages 7-21 …
9. Nirmala G, Anju R, Cost Minimization assignment problem using fuzzy quantifier, International Journal of Computer Science and Information Technologies, vol 5, no.79, pp 48 – 50, 2014.
10. Phani Bushan Rao P, Ravishankar N, Ranking fuzzy numbers with a distance method using circumcenter of centroids and an index of Modality, Advances in Fuzzy System, vol 10, no. 11, pp 55 – 61, 2011.
11. Ponnivalan K, Pathinathan T, Pentagonal Fuzzy Numbers, International Journal of Computing Algorithm, vol 3, pp 1003 – 1005, 2014.
12. Rajarajeswari P, Sahaya Sudha A, Karthika R, A New Operation On Hexagonal Fuzzy Number, International Journal of Fuzzy Logic Systems, vol 3, no. 3, pp 15 – 26, 2013.
13. Stefan Chanas, Dorota Kuchta, A concept of the optimal solution of the transportation problem with fuzzy cost coefficients vol. 83, pp. 299 – 305, 1996.
14. Supreet Kaur, Pentagonal Fuzzy Numbers, Journal of Emerging Technologies and Innovation Research, vol 5, no. 8, pp 330 – 333, 2018.
15. Valvis E, A new linear ordering of fuzzy numbers on subsets F(R), Fuzzy Optimization and Decision Making, vol 8, no.1, pp 41 – 63,2009.

16. Wang YJ, Lee HS, The revised method of ranking fuzzy numbers with an area between the centroid and original points, Computer and Mathematics with Applications, vol 55, no.20, pp 33 -42, 2008.
17. Yager R R, A procedure for ordering fuzzy subsets of the unit interval, Information Sciences, vol 24, no. 1, pp 43 – 61, 1981.
18. Zadeh L.A, Fuzzy Sets, Information and Control, vol. 8,no.3, pp.338-353,1965.