

Difference Square Mean Fuzzy Labelling of Graphs Constructed from Path Graph

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Abstract:

Computer science applications make extensive use of graph theory. Information mining, picture classification, grouping images, photo capture, and connectivity are particularly important study areas in computer science. Fuzzy labelling models provide better accuracy, adaptability, and interoperability to the framework than conventional fuzzy models. They are used in many areas of basic mathematics, including traditional computer science and physical scientific study. A common problem in mathematics is the distance properties in a graph. In fuzzy mean labelling graphs, paths have some exciting applications. Using lengths to securely identify the vertex in an organization is one such application. As the outcome, in this study, we explore four paths that are DSMFLs in graphs: comb graph, double comb, path-attached pendant vertex, and path-attached two pendant vertices. In fuzzy labelling graphs, there exist several DSMFLs. Whenever the force of connection between each pair of vertices G matches the value of the membership of the edges, G appears self-sufficient in relation to the DSMF. Furthermore, it is established that any interconnected fuzzy labelling graph is both a path graph and a comb graph.

MSC Classification:05C76, 05C78

Key Words: Difference Square Mean Fuzzy, Path graph, Pendant Vertex, Comb graph, Double comb graph.

1. Introduction:

Graph labelling serves as a milestone on the route to enlightening the real world of mathematics. The technique of graph identification originates with Alex Rosa's 1967 publication. Several writers subsequently developed various types of graph labelling [4,5]. Graph labelling is the technique of assigning labels, which are often represented by integers, to graph elements. A vertex labelling function on graph containing G is one that converts G 's set of vertices into a collection of labels [1,2]. A graph containing this function is referred to as a vertex-labelled graph. Graph labelling is essential in a variety of domains, including network communication, computer code theory, radar technology, astrophysics, crystalline science, and circuit structure[3,6].

Fuzzy sets are a novel mathematical framework that illustrates the occurrence of uncertainty in real-life problems. It was presented by Zadeh in 1965, among its principles were first proposed by other independent scientists, namely Rosenfeld, a Bhutani, and Battou, in the 1970s [7,9,11]. In "Someremarks

on fuzzy graphs," Bhattacharya introduced the connectivity ideas across fuzzy cutting vertices and fuzzy linkages [14,18]." They investigated many fuzzy equivalents of graph theoretic notions, including routes, loops, and completeness. A number of issues need to be addressed with fuzzy graphs [8,10,12].

Though it is still relatively new, it is rapidly expanding and has multiple applications in a wide range of industries [13,16]. Furthermore, research into fuzzy graphs has grown at exponential rates, within mathematical and in its various uses in technological and scientific fields. The flexible graph is an extension of the crisp graph [15,17]. As a result, it is inevitable that many attributes are comparable to crisp graphs, but they also diverge in numerous locations [19].

Basic definitions:

Fuzzy Labelling Graph:

A graph with $G = (\sigma, \mu)$ is termed a fuzzy labeling graph if $\sigma: V \rightarrow [0, 1]$ and $\mu: V \times V \rightarrow [0, 1]$ are bijective based and $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for any u, v .

Difference Mean Square Labelling:

If $\eta: V \rightarrow [0, 1]$ and $\gamma: V \times V \rightarrow [0, 1]$ are bijective and $\eta(u, v) < \eta(u) \wedge \eta(v)$ and

$$(u, v) = |(\eta(u) - \eta(v))^2 / 2| \text{ Forever } (u, v) \in E \text{ are all different.}$$

Definition:

The Double comb graph is symbolized by $P_l \odot V_1$. The P_n route graph consists of $(l-1)$ edges and n vertices. A Two pendant edge connects each vertex in the path, forming the graph. The Double comb graph contains $2l$ vertices and $2l-1$ edges.

Definition:

The comb graph is symbolized by $P_l \odot V_{1,2}$. The P_n route graph consists of $(l-1)$ edges and n vertices. A pendant edge connects each vertex $(1, 2)$ in the path, forming the graph. The comb graph contains $2l$ vertices and $2l-1$ edges.

Definition:

Comb is a graph formed by connecting every vertex along a route with a double pendent edge.

Definition:

$P_r(1, 2, \dots, n)$ is a graph formed by attaching l pendant vertices at each of its l^{th} vertices to form a path of vertices with length r .

Theorem: 1

A Path graph $P_r \odot k_2$ is a DSMFL graph.

Proof:

Let $P_r \odot k_2$ be the path related graph. The order and size, $g = 2r + 2s$, $h = 2r + 2s - 1$

Let $u_1, u_2, \dots, u_r$ be the path p_n and v_1, v_{i+1} be the vertices of k_2

Which are joined to the vertex v_i of path p_n , $1 \leq l \leq r$.

The resultant graph is $P_n \odot k_2$

Whose edge set is,

$$\gamma = \{v_i, v_{i+1}, / 1 \leq l \leq r\} \cup \{v_i, u_l, v_{i+1}, / 1 \leq l \leq r\}$$

For the vertices membership value

$$\eta: v \rightarrow [0,1]$$

$$\eta(v_1) = (1).01$$

$$\eta(v_{2l}) = (4r + 3s + 3 - 6l)0.01 ; \quad \left\{ \text{for } l = 1, 2, \dots, \frac{r}{2}, \text{ if } r \text{ is even} \right.$$

$$\left. \text{for } l = 1, 2, \dots, \frac{r-1}{2}, \text{ if } r \text{ is odd} \right\}$$

$$\eta(v_{2l+1}) = (2r + 3s - 1 + 6l)0.01 ; \left\{ \text{for } l = 1, 2, \dots, \frac{r-2}{2}, \text{ if } r \text{ is even} \right.$$

$$\left. \text{for } l = 1, 2, \dots, \frac{r-1}{2}, \text{ if } r \text{ is odd} \right\}$$

$$\eta(u_{4l-3}) = (5r + 4s + 1 - 6l)0.01 ; \left\{ \text{for } l = 1, 2, \dots, \frac{r+1}{2}, \text{ if } r \text{ is odd} \right.$$

$$\left. \text{for } l = 1, 2, \dots, \frac{r}{2}, \text{ if } r \text{ is even} \right\}$$

$$\eta(u_{4l-2}) = (5r + 3s + 1 - 6l).01 ;$$

$$\eta(u_{4l-1}) = (r - 2s + 3 + 6l)0.01 ; \quad \left\{ \text{for } l = 1, 2, \dots, \frac{r-1}{2}, \text{ if } r \text{ is odd} \right.$$

$$\left. \eta(u_{4l}) = (s - 2r + 5 + 6l)0.01 ; \quad \text{for } l = 1, 2, \dots, \frac{r}{2}, \text{ if } r \text{ is even} \right\}$$

Let edge set $\{e_1, e_2, \dots, e_l, e'_1, e'_2, e'_3, \dots, e'_l\}$

For the edges , membership value $\gamma: E \rightarrow [0,1]$

$$\gamma(e_l) = \left\{ \frac{|v_{2l} - v_{2l+1}|^2 0.01}{2} \right\} \text{ if } 1 \leq l \leq r$$

$$\gamma(e_l) = \left\{ \frac{|v_{2l} - u_{4l}|^2 0.01}{2} \right\} \text{ if } 1 \leq l \leq s.$$

$$\gamma(e'_l) = \left\{ \frac{|v_{2l} - u_{4l-1}|^2 0.01}{2} \right\} \text{ if } 1 \leq l \leq s.$$

$$\gamma(e'_l) = \left\{ \frac{|v_{2l+1} - u_{4l-2}|^2 0.01}{2} \right\} \text{ if } 1 \leq l \leq s.$$

$$\gamma(e'_l) = \left\{ \frac{|v_{2l+1} - u_{4l-3}|^2 0.01}{2} \right\} \text{ if } 1 \leq l \leq s.$$

Important condition, for Fuzzy Membership $\gamma(e_i) < \min(\eta(v_l), \eta(v_{l+1}))$

The above function f admits DSMFL graph

Hence, the graph $P_r \odot k_2$ is DSMFL graph..

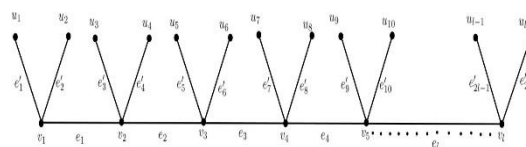


Fig. 1: DSMF graph of $P_n \odot k_2$

Example:

If n is Even

The graph $P_4 \odot k_2$ is shown in Fig.2.

$g = 12, h = 11$ the vertex labels are.

These Fuzzy Labels are satisfying DSMFL.

Hence the graph $P_4 \odot k_2$ is extra **DSMFL** graph.

Vertex Labels:

$$\begin{aligned}\eta(v_1) &= 0.01 & \eta(u_1) &= 0.23 \\ \eta(v_2) &= 0.19 & \eta(u_2) &= 0.21 \\ \eta(v_3) &= 0.07 & \eta(u_3) &= 0.03 \\ \eta(v_4) &= 0.13 & \eta(u_4) &= 0.05 \\ \eta(u_5) &= 0.17 & \eta(u_6) &= 0.15 \\ \eta(u_7) &= 0.09 & \eta(u_8) &= 0.11\end{aligned}$$

Edge Labels:

$$\begin{aligned}\gamma(e_1) &= 0.00162 & \gamma(e_1') &= 0.00242 \\ \gamma(e_2) &= 0.0072 & \gamma(e_2') &= 0.002 \\ \gamma(e_3) &= 0.0018 & \gamma(e_3') &= 0.0128 \\ \gamma(e_4') &= 0.0098 & & \\ \gamma(e_5') &= 0.005 & \gamma(e_6') &= 0.0032 \\ \gamma(e_7') &= 0.0008 & \gamma(e_8') &= 0.0002\end{aligned}$$

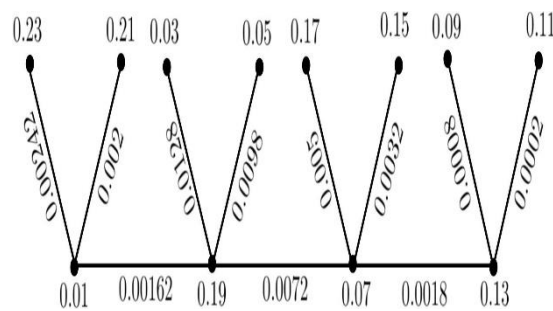


Fig. 2.

If n is Odd

The graph $P_5 \odot k_2$ is shown in Fig.3.

$g = 15, h = 14$ the vertex labels are.

These Fuzzy Labels are satisfying DSMFL.

Hence the graph $P_5 \odot k_2$ is extra **DSMFL** graph.

Vertex Labels:

$$\begin{aligned}\eta(v_1) &= 0.01 & \eta(u_1) &= 0.29 \\ \eta(v_2) &= 0.25 & \eta(u_2) &= 0.27 \\ \eta(v_3) &= 0.07 & \eta(u_3) &= 0.03 \\ \eta(v_4) &= 0.19 & \eta(u_4) &= 0.05 \\ \eta(v_5) &= 0.13 & & \\ \eta(u_5) &= 0.23 & \eta(u_6) &= 0.21 \\ \eta(u_7) &= 0.09 & \eta(u_8) &= 0.11 \\ \eta(u_9) &= 0.17 & \eta(u_{10}) &= 0.15\end{aligned}$$

Edge Labels :

$$\begin{aligned}\gamma(e_1) &= 0.00288 & \gamma(e_1') &= 0.00392 \\ \gamma(e_2) &= 0.0162 & \gamma(e_2') &= 0.00338\end{aligned}$$

$$\begin{aligned}
 \gamma(e_3) &= 0.0072 & \gamma(e_3') &= 0.0242 \\
 \gamma(e_4) &= 0.0018 & \gamma(e_4') &= 0.02 \\
 \gamma(e_5') &= 0.0128 & \gamma(e_6') &= 0.0098 \\
 \gamma(e_7') &= 0.005 & \gamma(e_8') &= 0.0032 \\
 \gamma(e_9') &= 0.0008 & \gamma(e_{10}') &= 0.0002
 \end{aligned}$$

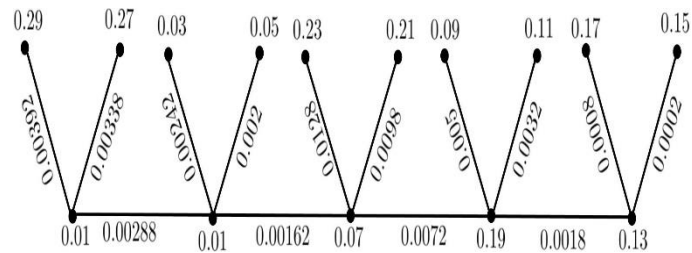


Fig. 3.

Theorem: 2:

Comb graph $p_r \odot K_{1,2}$ is a DSMFL graph for $r \geq 1, s = 2$.

Proof:

Let G be the comb graph $p_r \odot K_{1,2}$. The order of $G = 2r + 2s + 2$, and size of G , $h = 2r + 2s + 1$.

Let $\eta(G) = \{v_1, v_2, \dots, v_r, u_1, u_2, \dots, u_r, w_1, w_2, \dots, w_r\}$

Where u'_i 's are adjacent to v'_i 's, and w'_i 's.

The edge set

$\gamma(G) = \{e_1, e_2, \dots, e_l, f_1, f_2, \dots, f_l, g_1, g_2, \dots, g_l\}$.

Where $e_l = \{u_l, u_{l+1}\}$, $f_l = \{u_l, v_l\}$, $g_l = \{v_l, w_l\}$

Vertex fuzzy membership Function:

define $\eta: v \rightarrow [0,1]$

$$\begin{aligned}
 \eta(v_{2l-1}) &= (6r + 5s + 3 - 8l)0.01; & l &= 1, 2, \dots, \frac{r+1}{2} \text{ if } r \text{ is odd} \\
 & & l &= 1, 2, \dots, \frac{r}{2} \text{ if } r \text{ is even} \\
 \eta(v_{2l}) &= (s - r + 8l)0.01; & l &= 1, 2, \dots, \frac{r-1}{2} \text{ if } r \text{ is odd} \\
 & & l &= 1, 2, \dots, \frac{r}{2} \text{ if } r \text{ is even} \\
 \eta(u_{2l-1}) &= (s - 2r - 3 + 8l)0.01; & l &= 1, 2, \dots, \frac{r+1}{2} \text{ if } r \text{ is odd} \\
 & & l &= 1, 2, \dots, \frac{r}{2} \text{ if } r \text{ is even} \\
 \eta(u_{2l}) &= (6r + 4s - 1 - 8l)0.01; & l &= 1, 2, \dots, \frac{r-1}{2} \text{ if } r \text{ is odd} \\
 & & l &= 1, 2, \dots, \frac{r}{2} \text{ if } r \text{ is even} \\
 \eta(w_{4l-3}) &= (5r + 6s + 2 - 8l)0.01; & l &= 1, 2, \dots, \frac{r+1}{2} \text{ if } r \text{ is odd} \\
 \eta(w_{4l-2}) &= (5r + 5s + 2 - 8l)0.01; & l &= 1, 2, \dots, \frac{r}{2} \text{ if } r \text{ is even}
 \end{aligned}$$

$$\begin{aligned}\eta(w_{4l-1}) &= (s - 2r - 1 + 8l)0.01; & l = 1, 2, \dots, \frac{r-1}{2} \text{ if } r \text{ is odd} \\ \eta(w_{4l}) &= (s - 2r + 1 + 8l)0.01; & l = 1, 2, \dots, \frac{r}{2} \text{ if } r \text{ is even}\end{aligned}$$

Edge fuzzy membership Function:

define $\gamma: E \rightarrow [0, 1]$

$$\gamma(G) = \{e_l = \frac{|u_{2l} - u_{2l-1}|^2 0.01}{2}\} \quad \text{if } 1 \leq l \leq r$$

$$\gamma(G) = \{f_l = \frac{|v_{2l} - u_{2l}|^2 0.01}{2}\} \quad \text{if } 1 \leq l \leq r$$

$$\gamma(G) = \{f_{2l} = \frac{|v_{2l-1} - u_{2l-1}|^2 0.01}{2}\} \quad \text{if } 1 \leq l \leq r.$$

$$\gamma(G) = \{g_l = \frac{|u_{2l-1} - w_{2l-1}|^2 0.01}{2}\} \quad \text{if } 1 \leq l \leq r.$$

$$\gamma(G) = \{g_{2l} = \frac{|u_{2l} - w_{2l}|^2 0.01}{2}\} \quad \text{if } 1 \leq l \leq r.$$

Important Condition, for Fuzzy Membership $\gamma(e_i) < \eta(u_i) \wedge \eta(u_{i+1})$

The above function f admits DSMFL graph. Hence the graph $p_r \odot k_{1,2}$ is a DSMFL graph.

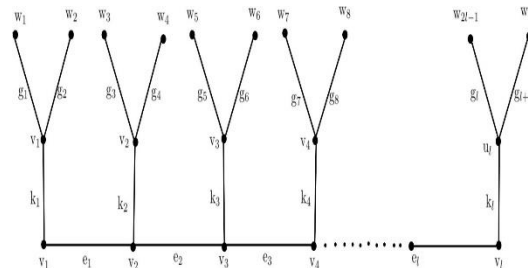


Fig.4: DSMF of $p_r \odot k_{1,2}$

Example:2

Comb graph $p_4 \odot K_{1,2}$ is a DSMFL graph for $r \geq 1, s = 2$ as shown in Fig.5.

$g=16, h=15$

Vertex Labels:

$$\begin{aligned}\eta(v_1) &= 0.31 & \eta(u_1) &= 0.01 \\ \eta(v_2) &= 0.07 & \eta(u_2) &= 0.25 \\ \eta(v_3) &= 0.23 & \eta(u_3) &= 0.09 \\ \eta(v_4) &= 0.15 & \eta(u_4) &= 0.17 \\ \eta(w_1) &= 0.29 & \eta(w_2) &= 0.27 \\ \eta(w_3) &= 0.03 & \eta(w_4) &= 0.05 \\ \eta(w_5) &= 0.21 & \eta(w_6) &= 0.19 \\ \eta(w_7) &= 0.11 & \eta(w_8) &= 0.13\end{aligned}$$

Edge Labels:

$$\begin{aligned}\gamma(e_1) &= 0.0288 & \gamma(e_2) &= 0.0128 \\ \gamma(e_3) &= 0.0032\end{aligned}$$

$$\begin{aligned}
 \gamma(k_1) &= 0.0045 & \gamma(k_2) &= 0.0162 \\
 \gamma(k_3) &= 0.0098 & \gamma(k_4) &= 0.0002 \\
 \gamma(g_1) &= 0.00392 & \gamma(g_2) &= 0.00338 \\
 \gamma(g_3) &= 0.0242 & \gamma(g_4) &= 0.02 \\
 \gamma(g_5) &= 0.0072 & \gamma(g_6) &= 0.005 \\
 \gamma(g_7) &= 0.0018 & \gamma(g_8) &= 0.0008
 \end{aligned}$$

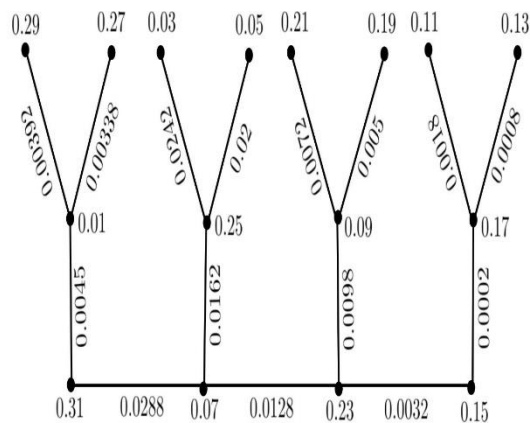


Fig.5: DSMT of $p_r \odot k_{1,2}$

Theorem: 3

Double comb graph $P_n \odot 2K_1$ is a DSMT graph.

Proof:

Let $P_r \odot 2K_1$ be the double comb graph.

The order and size of G are $g = 3r, h = 3r$

The Vertex set $\eta(G) = \{v, v_1, v_2, \dots, v_r, u, u_1, u_2, \dots, u_r, w, w_1, w_2, \dots, w_r\}$ where v adjacent vertex of v_1 and $u_1, u_2, \dots, u_r$ are adjacent vertices of $v_1, v_2, \dots, v_r$ and v adjacent vertex of w and $u_1, u_2, \dots, u_r$ are adjacent vertices of $w_1, w_2, \dots, w_r$.

The edge set

$$\eta(G) = \{e_1, e_2, \dots, e_r, e'_1, e'_2, \dots, e'_r, e''_1, e''_2, \dots, e''_r\}$$

Where $e_l = (u_l v_l) e''_l = (u_l w_l)$

$$e''_l = (u_l u_{l+1}) e_{11} = (u, v) e_{12} = (u, w) e_{13} = (u, u_1)$$

Define the function

Vertex membership function

Define $\eta: v \rightarrow [0, 1]$

$$\eta(u) = (1)0.01, \eta(v) = (6r - 1)0.01, \eta(w) = (6r - 3)0.01$$

$$\begin{aligned}
 \eta(v_l) &= \begin{cases} (3 + 3(l - 1))0.01; & 1 \leq l \leq r, l \equiv 1(mod 2) \\ (6r - 7 - 3(l - 2))0.01; & 1 \leq l \leq r, l \equiv 0(mod 2) \end{cases} \\
 \eta(u_l) &= \begin{cases} (6r - 5 - 3(l - 1))0.01; & 1 \leq l \leq r, l \equiv 1(mod 2) \\ (7 + 3(l - 2))0.01; & 1 \leq l \leq r, l \equiv 0(mod 2) \end{cases}
 \end{aligned}$$

$$\eta(w_l) = \begin{cases} (5 + 3(l - 1))0.01; & 1 \leq l \leq r, l \equiv 1(mod 2) \\ (6r - 9 - 3(l - 2))0.01; & 1 \leq l \leq r, l \equiv 0(mod 2) \end{cases}$$

The induced edge labels are

Define $\gamma: E \rightarrow [0,1]$

$$\gamma(e_{11}) = (3r - 1)0.01, \quad \gamma(e_{12}) = (3r - 2)0.01, \quad \gamma(e_{13}) = (3r - 3)0.01$$

$$\gamma(e_l) = \left\{ \frac{|v_l - u_l|^2 0.01}{2} \right\} \quad \text{if } 1 \leq l \leq r$$

$$\gamma(f_l) = \left\{ \frac{|u_l - w_l|^2 0.01}{2} \right\} \quad \text{if } 1 \leq l \leq r.$$

$$\gamma(k_l) = \left\{ \frac{|u_l - u_{l+1}|^2 0.01}{2} \right\} \quad \text{if } 1 \leq l \leq r.$$

The above function f admits DSMFL graph. Hence the graph $P_n \odot 2K_1$ DSMF graph

Important Condition , for Fuzzy Membership $\gamma(e_i) < \eta(u_i) \wedge \eta(u_{i+1})$.

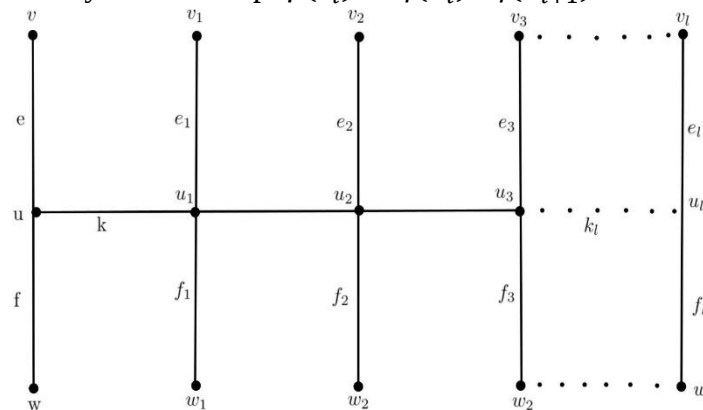


Fig 6: DSMFL of $P_n \odot 2K_1$

Example: 3

The graph $P_n \odot 2K_1$ is shown in Fig 7. The order and size are 15 and 14 respectively.

The vertex labels are,

Vertex Labels:

$$\eta(v_1) = 0.29 \quad \eta(u_1) = 0.01$$

$$\eta(v_2) = 0.03 \quad \eta(u_2) = 0.25$$

$$\eta(v_3) = 0.23 \quad \eta(u_3) = 0.27$$

$$\eta(v_4) = 0.09 \quad \eta(u_4) = 0.19$$

$$\eta(v_5) = 0.17 \quad \eta(u_5) = 0.13$$

$$\eta(w_1) = 0.27 \quad \eta(w_2) = 0.05$$

$$\eta(w_3) = 0.21 \quad \eta(w_4) = 0.11$$

$$\eta(w_5) = 0.15$$

Edge Labels:

$$\gamma(e_1) = 0.0039 \quad \gamma(f_1) = 0.00338$$

$$\gamma(e_2) = 0.0242 \quad \gamma(f_2) = 0.02$$

$$\gamma(e_3) = 0.0008 \quad \gamma(f_3) = 0.0018$$

$$\gamma(e_4) = 0.005 \quad \gamma(f_4) = 0.0032$$

$$\gamma(e_5) = 0.0008 \quad \gamma(f_5) = 0.0002$$

$$\gamma(k_1) = 0.0288 \quad \gamma(k_2) = 0.0002$$

$$\gamma(k_3) = 0.00032$$

$$\gamma(k_4) = 0.0018$$

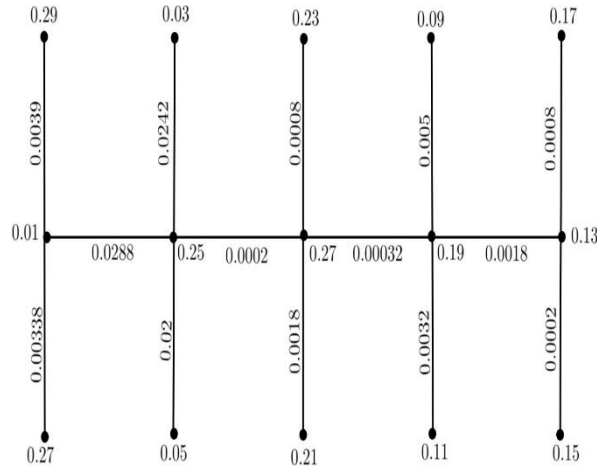


Fig 7: SDML of $P_n \odot 2K_1$

Theorem: 4

$P_{r-1}(1, 2, \dots, r)$ is a DSMF graph for $r \geq 1$

Proof:

Let $P_{r-1}(1, 2, \dots, r)$ be the path length $r - 1$ by joining i pendent vertices. The order

and size of G are $g = \frac{r^2+3r}{2}$, $h = \frac{r^2+3r-2}{2}$

The vertex set

$\eta(G) = \{u_1, u_2, \dots, u_n, v_{11}, v_{21}, v_{22}, \dots, v_{rr}\}$ where, u adjacent vertex of $v, u_1, u_2, \dots, u_r$ are adjacent vertices of $v_{l1}, v_{l2}, \dots, v_{lr}$. The edge set

$$\gamma(G) = \{e_1, e_2, \dots, e_r, e'_1, e'_2, \dots, e'_r, e\}$$

Where $e'_l = (u, u_l)$, $e'_l = (u_l, v_{lk})$, $e = (u, v)$

Define the function

Vertex membership function

Define $\eta: v \rightarrow [0, 1]$

$$\eta(u_{lk}) = \begin{cases} \left(3r + 2 - \frac{[l + 2l - 1]}{2}\right) 0.01; 1 \leq l \leq r; l \equiv 1 \pmod{2} \\ \left(\frac{2 + 2l - 2}{2}\right) 0.01; 1 \leq l \leq r; l \equiv 0 \pmod{2} \end{cases}$$

$$\eta(v) = (1)0.01$$

$$\eta(v_{lk}) = \begin{cases} \left(3r + 2 + \frac{[2 - 2 - 4k]}{2}\right) 0.01; 2 \leq l \leq r; l \equiv 0 \pmod{2}; 2 \leq k \leq l \\ \left(\frac{2 - 3 + 4k}{2}\right) 0.01; 2 \leq l \leq r; l \equiv 1 \pmod{2}; 2 \leq k \leq l \end{cases}$$

The edge labels are

The induced edge labels are

Define $\gamma: e \rightarrow [0, 1]$

$$\gamma(e_l) = \left\{\frac{|v_l - u_l|^2 0.01}{2}\right\} \quad \text{if } 1 \leq l \leq r$$

$$\gamma(f_{lk}) = \left\{ \frac{|u_l - u_{l+1}|^2 \cdot 0.01}{2} \right\} \quad \text{if } 1 \leq k \leq l.$$

$$\gamma(e_{11}) = |3r + r^2 - 2|$$

The above the function f admits DSMFL graph

Hence the graph $P_{r-1}(1, 2, \dots, r)$ are DSMF graph.

Important Condition, for Fuzzy Membership $\gamma(e_i) < \eta(u_i) \wedge \eta(u_{i+1})$.

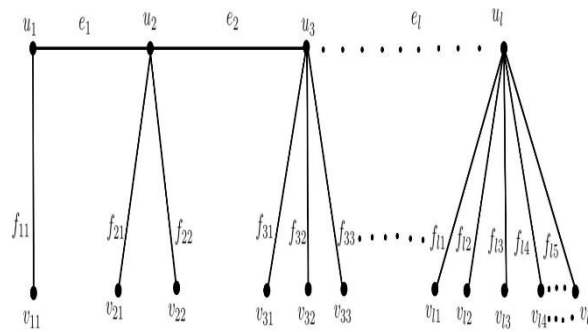


Fig 8: DSMFL of $P_{n-1}(1, 2, \dots, n)$

Example: 4

The graph P_4 is shown in Fig 4.6. The order and size are 20 and 19

The vertex labels are,

Vertex Labels:

$$\eta(u_1) = 0.39 \quad \eta(u_2) = 0.03$$

$$\eta(u_3) = 0.33 \quad \eta(u_4) = 0.11$$

$$\eta(u_5) = 0.23$$

$$\eta(v_{11}) = 0.01$$

$$\eta(v_{21}) = 0.037 \quad \eta(v_{22}) = 0.35$$

$$\eta(v_{31}) = 0.05 \quad \eta(v_{32}) = 0.07$$

$$\eta(v_{33}) = 0.09$$

$$\eta(v_{41}) = 0.31 \quad \eta(v_{42}) = 0.29$$

$$\eta(v_{43}) = 0.27 \quad \eta(v_{44}) = 0.25$$

$$\eta(v_{51}) = 0.13 \quad \eta(v_{52}) = 0.15$$

$$\eta(v_{53}) = 0.17 \quad \eta(v_{54}) = 0.19$$

$$\eta(v_{55}) = 0.21$$

Edge Labels:

$$\gamma(e_1) = 0.00648 \quad \gamma(e_2) = 0.0045$$

$$\gamma(e_3) = 0.0242 \quad \gamma(e_4) = 0.0072$$

$$\gamma(f_{11}) = 0.00722$$

$$\gamma(f_{21}) = 0.00578 \quad \gamma(f_{22}) = 0.00512$$

$$\gamma(f_{31}) = 0.0392 \quad \gamma(f_{32}) = 0.0338$$

$$\gamma(f_{33}) = 0.0288$$

$$\gamma(f_{41}) = 0.02 \quad \gamma(f_{42}) = 0.0162$$

$$\gamma(f_{43}) = 0.0128 \quad \gamma(f_{44}) = 0.0098$$

$$\begin{aligned}\gamma(f_{51}) &= 0.005 & \gamma(f_{52}) &= 0.0032 \\ \gamma(f_{53}) &= 0.0018 & \gamma(f_{54}) &= 0.0008 \\ \gamma(f_{55}) &= 0.0002\end{aligned}$$

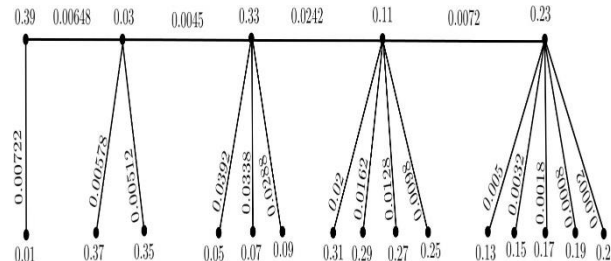


Fig 9: DSMFL of P_{n-1} (1, 2, ..., n)

2. Conclusion:

Fuzzy graph theory is increasingly being used in real-time system modeling, where the natural information provided by the structure changes with different levels of accuracy. Fuzzy modeling techniques aim to reduce the variances across the traditional mathematical models used in the fields of science and structures of construction and the graphical frameworks utilized in trained systems.

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